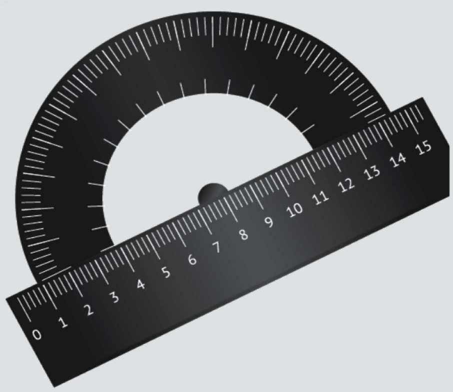
JEE Mains + Advance

MATHS

JEE Sarthi KOTA

Sample Module



Chapter

Function

JEE Module Details

(Total = 24)

CLASS - XI : 12 MODULES

PHYSICS

Module - 1	
Ch. No.	Chapter Name
1.	Mathematical Tools
2.	Vector
3.	Unit, Dimension and Measurement
4.	Kinematics
5.	Newton's Laws of Motion

Module - 2	
Ch. No.	Chapter Name
1.	Work Power and Energy
2.	Center of Mass & Collision
3.	Rotational Motion
4.	Gravitation

Module - 3	
Ch. No.	Chapter Name
1.	Fluid Mechanics
2.	Surface Tension
3.	Elasticity & Viscosity
4.	Simple Harmonic Motion

Module - 4	
Ch. No.	Chapter Name
1.	Thermometry & Calorimetry
2.	Thermal Expansion
3.	Kinetic Theory of Gases
4.	Thermodynamics
5.	Heat Transfer

CHEMISTRY

Module - 1	
Ch. No.	Chapter Name
1.	Some Basic Concept of Chemistry
2.	Atomic Structure
3.	Redox Reactions
4.	States of Matter

Module - 2	
Ch. No.	Chapter Name
1.	Chemical Equilibrium
2.	Ionic Equilibrium
3.	Chemical Thermodynamics & Energetics

Module - 3	
Ch. No.	Chapter Name
1.	Periodic Table and Periodic Properties
2.	Chemical Bonding
3.	Hydrogen and its compounds
4.	s-Block elements
5.	p-Block (13 to 14 groups)

Module - 4	
Ch. No.	Chapter Name
1.	IUPAC
2.	Isomerism
3.	GOC-I
4.	Hydrocarbons
5.	Environmental Chemistry

MATHEMATICS

Module - 1	
Ch. No.	Chapter Name
1.	Set & Relations
2.	Trigonometric Ratios
3.	Trigonometric Equation
4.	Solution of a Triangle

Module - 2	
Ch. No.	Chapter Name
1.	Sequence and Series
2.	Quadratic Equations and Inequalities
3.	Complex Numbers
4.	Limits & Derivative

Module - 3	
Ch. No.	Chapter Name
1.	Binomial Theorem
2.	Permutations and Combinations
3.	Straight Lines
4.	Circle

Module - 4	
Ch. No.	Chapter Name
1.	Parabola
2.	Hyperbola
3.	Ellipse

JEE : Mathematics

Sample Module

STUDENT NAME:____

SECTION: ____________________________ROLL NO: _____________

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❖ PREFACE ❖

This module covers the theoretical concepts associated with NEET syllabus and contain sufficient multiple choice and previous year questions. We are confident that students would find this module helpful for their preparations.

Research & Development team of NEET Sarthi keeps working to improve the study material. Suggestions and inputs from students and readers are always welcome.

About NEET Sarthi

NEET Sarthi, A platform for JEE, NEET, NTSE and other competitive exams, is an initiative by highly renowned faculties from Kota (the coaching capital of India) and Tech team from Bangalore (the Silicon Valley of India). Our mission is to provide extensive and high-quality education to students. Specially, our vision is to be top most institute in terms of academic quality & student care.

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"If you can dream it, you can do it" **-Dr. A.P.J. Abdul kalam**

Chapter-01
Functions

- **Definition**
- **Domain, Co-Domain & Range**
- **Algebraic Operations**
- **Methods of Determining Range**
- **Important Types of Functions**
- **Domains and Ranges of Common Functions**
- **Equal or Identical Functions**
- **Classification of Fucntions**
- **Composite of Uniformly and Non-Uniformly Defined Functions**
- **Homogeneous Functions**
- **Bounded Functions**
- **Implicit & Explicit Functions**
- **Odd & Even Functions**
- **Periodic Functions**
- **General**
- **Inverse of A Functions**
- **Some Graphical Transformations**

1. DEFINITION:

Let A and B be two sets and there exists a rule or manner or correspondence 'f' which associates each element of A, a unique element in B. Then f is called a function or mapping from A to B. It is denoted by the symbol

$$f : A \to B \text{ or } A \xrightarrow{f} B$$

which reads 'f 'is a function from A to B'or 'f maps A to B,

If an element a $\in$ A is associated with an element b $\in$ B then b is called 'the f image of a'or 'image of a under f 'or 'the value of the function 'f ' at a'. Also 'a' is called the pre-image of b or argument of b under the function f. We write it as b = f (a) or f: a $\to$ b or f: (a, b)

For example, let A $\equiv$ {–1, 0, 1} and B $\equiv$ {0, 1, 2,3}.

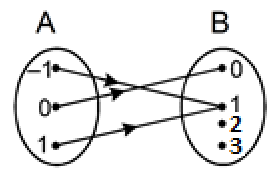
Then A × B $\equiv$ {(–1, 0), (–1, 1), (–1, 2), (–1,3), (0, 0), (0, 1), (0, 2), (0,3), (1, 0), (1, 1), (1, 2), (1,3)}

Now, " f: A $\to$ B defined by f(x) = x^2 " is the function such that

f $\equiv$ {(–1, 1), (0, 0), (1, 1)}

f can also be shown diagramatically by following mapping.

A function 'f 'from a set A to a set B is a subset of A× B



Note:

(1) Every function is a relation but every relation is not necessarily a function.

(2) A function is also called a mapping.

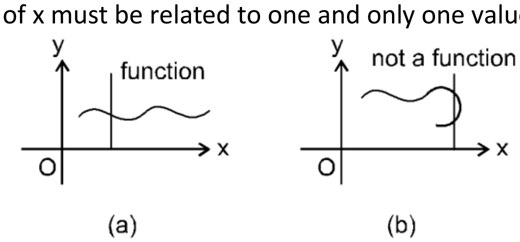
Note:

Every function say y = f(x): A $\to$ B. Here x is independent variable which takes its values from A while 'y' takes its

value from B. A relation will be a function if and only if

(i) x must be able to take each and every value of A and

(ii) one value of x must be related to one and only one value of y in set B.



Graphically: If any vertical line cuts the graph at more than one point, then the graph does not represent a function.

Every function from $A \to B$ satisfies the following conditions.

(i) $f \subset A \times B$

(ii) $\forall\ a \in A \Rightarrow (a, f(a)) \in f$ and

(iii) $(a, b) \in f$ & $(a, c) \in f \Rightarrow b = c$

2. DOMAIN, CO–DOMAIN & RANGE OF A FUNCTION:

Let $f: A \to B$, then the set A is known as the domain of f & the set B is known as co-domain of f. The set of all f images of elements of A is known as the range of f. Thus:

Domain of f = $\{a \mid a \in A, (a, f(a)) \in f\}$

Range of f = $\{f(a) \mid a \in A, f(a) \in B, (a, f(a)) \in f\}$

It should be noted that range is a subset of co–domain. If only the rule of function is given then the domain of the function is the set of those real numbers, where function is defined. For a continuous function, the interval from minimum to maximum value of a function gives the range.

3. ALGEBRAIC OPERATIONS ON FUNCTIONS:

Let f and g be function with domain D_1 and D_2 then the functions f + g, f – g, fg, f/g are defined as

$(f + g)(x) = f(x) + g(x)$; Domain $D_1 \cap D_2$

$(f - g)(x) = f(x) - g(x)$; Domain $D_1 \cap D_2$

$(f g)(x) = f(x) \cdot g(x)$; Domain $D_1 \cap D_2$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$; Domain = $\{x \in D_1 \cap D_2 \mid g(x) \neq 0\}$

4. METHODS OF DETERMINING RANGE:

(i) Representing x in terms of y

If $y = f(x)$, try to express as $x = g(y)$, then domain of $g(y)$ represents possible values of y, which is range of $f(x)$.

SOLVED EXAMPLES

Ex.1 **(i)** Which of the following correspondences can be called a function?

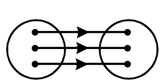
(A) $f(x) = x^3$; $\{-1, 0, 1\} \to \{-1, 0, 1, 2, 3\}$

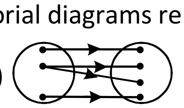
(B) $f(x) = \pm\sqrt{x}$; $\{0, 1, 4\} \to \{-2, -1, 0, 1, 2\}$

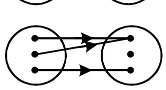
(C) $f(x) = \sqrt{x}$; $\{0, 1, 4\} \to \{-2, -1, 0\}$

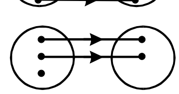
(D) $f(x) = -\sqrt{x}$; $\{0, 1, 4\} \to \{-2, -1, 0, 1, 2\}$

(ii) Which of the following pictorial diagrams represent the function

(A) 

(B) 

(C) 

(D) 

Sol. **(i)** f(x) in (A) and (D) are functions as definition of function is satisfied. while in case of (C) the given relation is a function, as each element of 1st set is related to unique element of 2nd set. Hence definition of function is not satisfied. While in case of (B), the given relation is not a function, as f(1) = ± 1 and f(4) = ± 2 i.e. element 1 as
well as 4 in 1st set is related with two elements of 2nd set. Hence definition of function is not satisfied.

(ii) A and C. In (D) one element of domain has no image, while in (B) one element of 1st set has two images in 2nd Set

Ex.2 Find the domain of following functions:

(i) $f(x) = \sqrt{x^2-9}$ **(ii)** log (2x – 6) **(iii)** $f(x) = \sqrt{x+3}-\sqrt{16-x^2}$

Sol. **(i)** $f(x) = \sqrt{x^2-9}$ is real iff $x^2 - 9 \geq 0$

$\Rightarrow |x| \geq 3 \quad \Rightarrow x \leq -3$ or $x \geq 3$

$\therefore$ The domain of f is $(-\infty, -3] \cup [3, \infty)$

(ii) log (2x – 6) is defined if

$2x - 6 > 0$

$2x \geq 6$

$x \geq 3$

Hence $x \in [3, \infty)$

(iii) $\sqrt{x+3}$ is real if $x + 3 \geq 0 \Leftrightarrow x \geq -3$

$\sqrt{16-x^2}$ is real if $16 - x^2 \geq 0 \Leftrightarrow -4 \leq x \leq 4$.

Thus the domain of the given function is

$\{x : x \in [-3, \infty) \cap [-4, 4] = [-3, 4]$

Ex.3 Find the range of $f(x) = \dfrac{x^2+x+1}{x^2+x-1}$

Sol. $f(x) = \dfrac{x^2+x+1}{x^2+x-1}$ $\{x^2 + x + 1$ and $x^2 + x - 1$ have no common factor$\}$

$y = \dfrac{x^2+x+1}{x^2+x-1}$

$\Rightarrow yx^2 + yx - y = x^2 + x + 1$

$\Rightarrow (y - 1) x^2 + (y - 1) x - y - 1 = 0$

If y = 1, then the above equation reduces to –2 = 0. Which is not true.

Further if $y \neq 1$, then $(y - 1) x^2 + (y - 1) x - y - 1 = 0$ is a quadratic and has real roots if

$(y - 1)^2 - 4 (y - 1) (-y - 1) \geq 0$

i.e. if $y \leq -3/5$ or $y \geq 1$ but $y \neq 1$

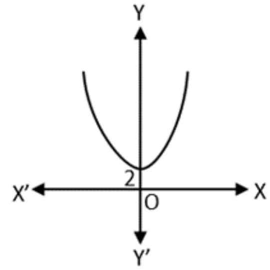
Thus the range is $(-\infty, -3/5] \cup (1, \infty)$

(ii) Graphical Method :

The set of y– coordinates of the graph of a function is the range.

EX. 4 Find the range of $f(x) = x^2 + 2$

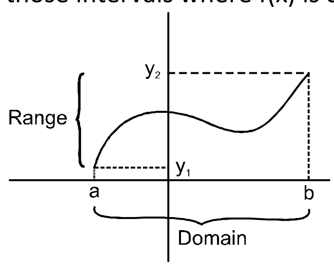
Sol. $f(x) = x^2 + 2$



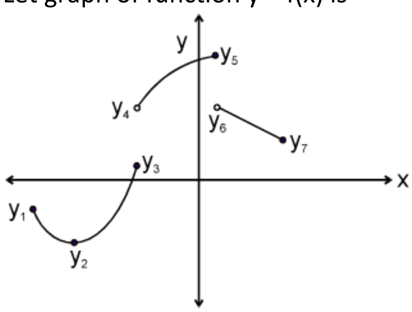
Domain = R

Range = $(2, \infty)$

Further if f(x) happens to be continuous in its domain then range of f(x) is [min f(x), max. f(x)]. However, for sectionally continuous functions, range will be union of [min f(x), max. f(x)] over all those intervals where f(x) is continuous, as shown by following example.



Ex.5 Let graph of function y = f(x) is



Then range of above sectionally continuous function is $[y2, y3] \cup (y4, y5] \cup (y7, y6]$

(iii) Using monotonocity:

Many of the functions are monotonic increasing or monotonic decreasing. In case of monotonic continuous functions, the minimum and maximum values lie at end points of domain. Some of the common function which are increasing or decreasing in the interval where they are **continuous** is as under.

For monotonic increasing functions in [a, b]

(i) $f'(x) \geq 0$

(ii) Range is [f(a), f(b)]

For monotonic decreasing functions in [a, b]

(i) $f'(x) \leq 0$

(ii) Range is [f(b), f(a)]

Ex.6 Find the range of following functions

(i) $f(x) = \log_3\{\log_{1/2}(x^2 + 4x + 4)\}$

(ii) $f(x) = \sin^2x - 5\sin x - 6$.

Sol. **(i)** $f(x) = \log_3\{\log_{1/2}(x^2 + 4x + 4)\}$

Firstly, finding the domain

$\log_{1/2}(x^2 + 4x + 4) > 0$

$x^2 + 4x + 4 < 1 \Rightarrow x^2 + 4x + 3 < 0$

$\Rightarrow (x+1)(x+3) < 0 \Rightarrow -3 < x < -1$

Also, $x^2 + 4x + 4 > 0$

$(x+2)^2 > 0 \Rightarrow x \neq -2$

Hence, $x \in (-3, -1) \{-2\}$

Since $0 < \log_{1/2}(x^2 + 4x + 4) < \infty \forall x \in$ domain thus

Range $\in R$

(ii) $f(x) = \sin^2 x - 5\sin x - 6 = \sin^2 x - 2$

$\sin^2 x - 2\left(\frac{5}{2}\right)\sin x + \frac{25}{4} - 6 - \frac{25}{4}$

$= \left(\sin x - \frac{5}{2}\right)^2 - \frac{49}{4}$

Where $\frac{9}{4} \leq \left(\sin x - \frac{5}{2}\right)^2 \leq \frac{49}{4}$

Hence, $f(x) \in [-10, 0]$. Ans.

Ex.7 If $f(x) = \frac{x}{x-1} = \frac{1}{y}$, then f (y) equals

(1) x (2) x – 1 (3) x + 1 (4) 1 – x

Sol. $f(y) = \frac{y}{y-1} = \frac{(x-1)/x}{\frac{x-1}{x} - 1} = \frac{x-1}{x-1-x} = 1 - x.$

Ans. **[4]**

Ex.8 The domain of $f(x) = \frac{1}{x^3 - x}$ is -

(1) R – {–1,0,1} (2) R (3) R – {0,1} (4) None of these

Sol. Domain = $\{x; x \in R; x^3 - x \neq 0\}$
$= R - \{-1, 0, 1\}$

Ex.9 The range of $f(x) = \cos \frac{\pi[x]}{2}$ ([.] represents G.I.F.) is -

(1) {0,1} (2) {–1,1} (3) {–1,0,1} (4) [–1,1]

Sol. [x] is an integer, cos (–x) = cos x and

$\cos\left(\frac{\pi}{2}\right) = 0$, $\cos 2\left(\frac{\pi}{2}\right) = -1.$

$\cos 0\left(\frac{\pi}{2}\right) = 1$, $\cos 3\left(\frac{\pi}{2}\right) = 0$ ... (2)

Hence range = {–1,0,1}

Ans. [3]

PRACTICE SECTION-01

Q.1 If X = {a, b, c, d, e} & Y = {$\alpha, \beta, \gamma, \delta, \theta$} then which of the following subset(s) of X × Y is/are a function from X to Y.

(1) {(a, γ) (b, γ) (b, δ) (d, θ) (e, β) (c, β)}
(2) {(a, γ) (b, α) (c, θ) (d, β)}
(3) {(a, α) (b, θ) (c, γ) (d, δ) (e, β)}
(4) {(a, γ) (b, γ) (c, γ) (d, γ) (e, γ)}

Q.2 Find the domain of following functions.

(i) $f(x)=\sqrt{x^2-x-6}+\sqrt{6-x}$
(ii) $f(x)=\sqrt{3x-x^2}$

Q.3 Find domain of the function

(i) $f(x)=\log_3\left(\log_{1/3}\left(x^2+10x+25\right)\right)+\dfrac{1}{[x]+5}$.

(where [.] denotes greatest integer function)

Q.4 Which of the following is a function?

(1) {(2,1), (2,2), (2,3), (2,4)}
(2) {(1,4), (2,5), (1,6) , (3,9)}
(3) {(1,2), (3,3), (2,3), (1,4)}
(4) { (1,2), (2,2), (3,2), (4,2)}

Q.5 Function $f(x) = x^{-2} + x^{-3}$ is -

(1) a rational function (2) an irrational function (3) an inverse function (4) None of these

Q.6 The domain of function $f(x) = \sqrt{2^x-3^x}$ is -

(1) $(-\infty, 0]$ (2) R (3) $[0, \infty)$ (4) No value of x

Q.7 The range of function $f(x) = \dfrac{x^2}{1+x^2}$ is -

(1) R – {1} (2) $R^+ \cup \{0\}$ (3) [0, 1] (4) None of these

Q.8 If f : R $\to$ R. f (x) = 2x + |x| , then
f (3x) – f (–x) – 4x equals -

(1) f(x) (2) – f(x) (3) f (–x) (4) 2f(x)

ANSWER KEY					
Q.1	**Q.4**	**Q.5**	**Q.6**	**Q.7**	**Q.8**
3,4	4	1	1	4	4

Q.2 **(i)** $(-\infty, -2] \cup [3, 6]$
(ii) [0,3]

Q.3 (i) Domain of f(x) $\in (-6, -5)$

5. IMPORTANT TYPES OF FUNCTIONS:

(i) Polynomial function:

If a function f is defined by f (x) = $a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + ... + a_{n\_1} x + a_n$ where n is a non negative integer and $a_0, a_1, a_2, ..., a_n$ are real numbers and $a_0 \neq 0$, then f is called a polynomial function of degree n. Here a_0 is leading coefficient.

E.g. $x^3 + 5x + 5$

Note:

- A polynomial function is always continuous.
- There are two polynomial functions, satisfying the relation;
  f(x).f(1/x) = f(x) + f(1/x). They are:

(i) $f(x) = x^n + 1$ & **(ii)** $f(x) = 1 - x^n$, where n is a positive integer.

- A polynomial of odd degree has its range $(-\infty, \infty)$ but a polynomial of degree even has a range which is always subset of R

(ii) Algebraic Function

A function f is called an algebraic function if it can be constructed using algebraic operations such as addition, substraction, multiplication, division and taking roots, started with polynomials.

e.g. $f(x) = \sqrt{x^2 + 1}$

Note that all polynomial are algebraic but the converse is not true. Functions which are not algebraic are known as transcedental function.

(iii) Rational Function:

A rational function is a function of the form. y = f (x) = $\frac{g(x)}{h(x)}$, where g (x) & h (x) are polynomials & h (x) $\neq$ 0.

The domain of f (x) is set of real x such that h (x) $\neq$ 0.

$f(x) = \frac{2x^2 + x + 1}{x^2 - 4}$;D = {x | x $\neq \pm$ 2}

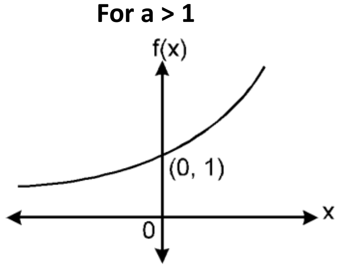
(iv) Exponential Function:

A function $f(x) = a^x = e^{x \ln a}$ (a > 0, a $\neq$ 1, x $\in$R) is called an exponential function. f (x) = a^x is called an exponential function because the variable x is the exponent. It should not be confused with power function. g (x) = x^2 in which variable x is the base.

Note: For f (x) = e^x domain is R and range is R^+

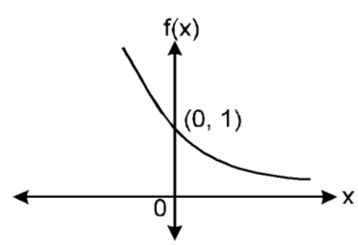
Case - I

For a > 1



Case - II

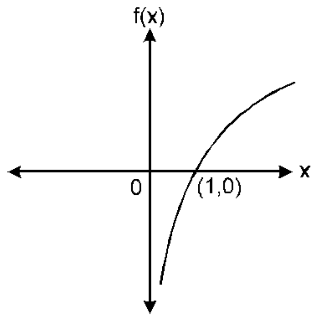
For 0 < a < 1



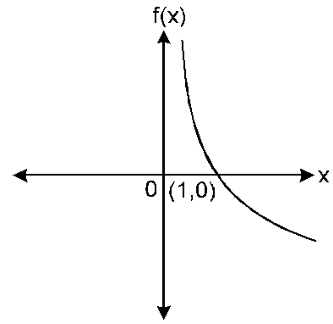
(v) Logarithmic Function:

f(x) = logax is called logarithmic function, where a > 0 and a $\neq$ 1 and x > 0. Its graph can be as follows

Case- I
For a > 1

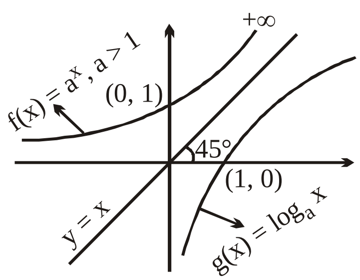


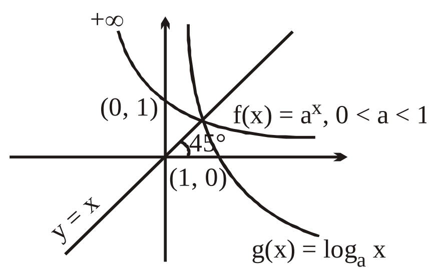
Case- II
For 0 < a < 1



Note:

Exponential and logarithmic functions are inverse of each other. Graph of exponential function can be obtained by taking mirror image of exponential curve on y = x.





Properties of Logarithmic Functions:

Let f and g be two positive real valued functions and $0 < a \neq 1$ be any real number. Then the following properties hold good

(i) $\log_a(f(x)g(x)) = \log_a f(x) + \log_a g(x)$

(ii) $\log_a\left(\frac{f(x)}{g(x)}\right) = \log_a f(x) - \log_a g(x)$

(iii) $\log_a(f(x))^{2n} = 2n\log_a|f(x)|$

(iv) $\log_{a^m}(f(x))^n = \frac{n}{m}\log_a f(x) = \log_{a^{(1/n)}}(f(x))^{1/m}$

(v) $(f(x))^{\log_a g(x)} = (g(x))^{\log_a f(x)}$ and $a^{\log_a f(x)} = f(x)$

Logarithmic Inequality:

1. If $\log_a x \geq m$ then within the domain

   (i) $x \geq a^m$ if $a > 1$

   (ii) $x \leq a^m$ if $0 < a < 1$

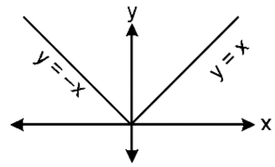
2. If $\log_a x \leq m$ then within domain

   (i) $x \leq a^m$ if $a > 1$

   (ii) $x \geq a^m$ if $0 < a < 1$

(vi) Modulus Function/Absolute Value Function:

A function y = f (x) = $|x|$ is called the absolute value function or Modulus function. It is defined as : y = $|x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$



y = |x|

(i) If a $\ge$ 0 and $|x| \le$ a , then -a $\le$ x $\le$ a, if a < 0 and $|x|$ < a, then no value of x exists. $|x| \le$ a $\Rightarrow$ x $\le$ -a or x $\ge$ a; a $\ge$ 0. If a $\le$ 0 and $|x| \ge$ a, then x $\in$ R.

(ii) $|x \pm y| \le |x| + |y| \ \forall$ x,y $\in$ R and $|x+y| = |x| + |y|$ if xy $\ge$ 0.

$|x-y| = |x| - |y| \Rightarrow \ge 0$ and $y \ge 0$ or $x \le 0$ and $y \le 0$ and $|x| \ge |y|$.

(iii) $|x+y| \ge ||x| - |y||$

(iv) The minimum value of $|x-a| + |x-b| = |a-b|$

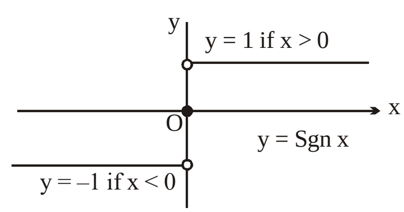
(v) If $|x-a| + |x-b| = |a-b| \Rightarrow a \le x \le b$

(vi) If $a \le |x| \le b$ and a, $b \ge 0$ then $x \in [-b,-a] \cup [a,b]$

(vii) The maximum value of $||x-a| - |x-b|| = |a-b|$

(vii) Signum Function:

A function y= f (x) = Sgn (x) is defined as follows :



y = f (x) = $\begin{cases} 1 & \text{for} \quad x > 0 \\ 0 & \text{for} \quad x = 0 \\ -1 & \text{for} \quad x < 0 \end{cases}$

It is also written as sgn x = $\begin{cases} \dfrac{|x|}{x}; & x \ne 0 \\ 0; & x = 0 \end{cases}$

Note: sgn f(x) = $\begin{cases} \dfrac{|f(x)|}{f(x)}; & f(x) \ne 0 \\ 0; & f(x) = 0 \end{cases}$

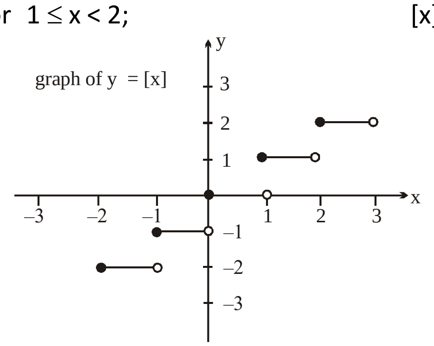
(viii) Greatest Integer or Step Up Function:

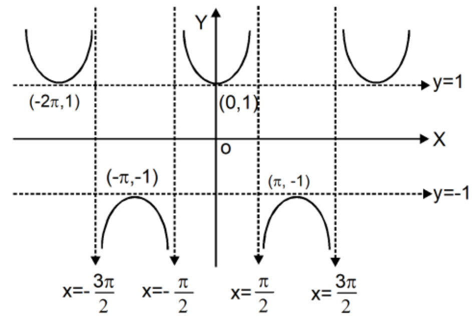
The function y = f (x) = [x] is called the greatest integer function where [x] denotes the greatest integer less than or equal to x.

Note that for:

for $-2 \le x < -1$; [x] = – 2; for $-1 \le x < 0$; [x] = – 1;

for $0 \le x < 1$; [x] = 0 for $1 \le x < 2$; [x] = 1; and so on.





Properties of Greatest Integer Function:

(a) $[x] \le x < [x]+1$ and $x-1 < [x] \le x$, $0 \le x-[x] < 1$

(b) $[x \pm m] = [x] \pm m$ if m is an integer.

(c) $[x]+[y] \le [x+y] \le [x]+[y]+1$

(d) $[x]+[-x]=\begin{cases} 0, \text{if } x \text{ is an integer} \\ -1, \text{otherwise} \end{cases}$

(ix) Fractional Part Function:

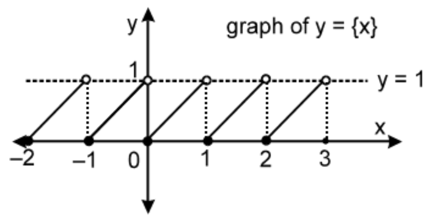
It is defined as:

$g(x) = \{x\} = x - [x]$.

e.g. the fractional part of the no. 2.1 is

$2.1 - 2 = 0.1$ and the fractional part of -3.7 is 0.3.

The period of this function is 1 and graph of this function is as shown.



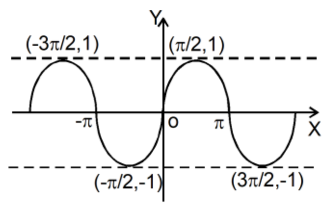
(X) Trigonometric Functions:

(a) Sine function

f(x) = sin x

Domain = R

Range = [–1, 1].

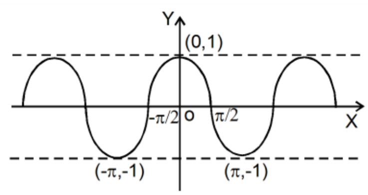


(b) Cosine function

f(x) = cos x

Domain = R

Range = [–1, 1].

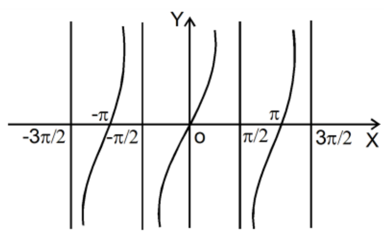


(c) Tangent function

f(x) = tan x

Domain = $R - \left\{\frac{(2n+1)\pi}{2}\right\}$

Range = R.

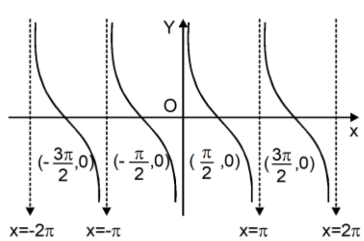


(d) Cotangent function

f(x) = cot x

Domain = R – {nπ: n ∈ Z};

Range = R.



(e) Cosecant function

f(x) = cosec x

Domain = R – {nπ : n ∈ Z};

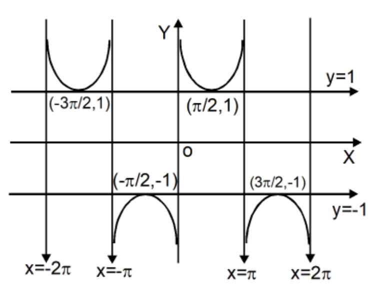
Range = R – (–1, 1)

(f) Secant function

f(x) = sec x

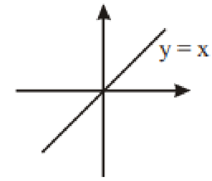
Domain = R – {(2n + 1) π/2 : n ∈ Z};

Range = R – (–1, 1)



(XI) Identity function:

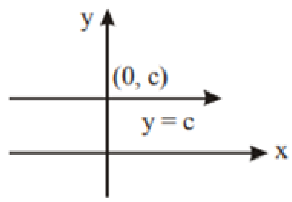
The function f : A→A defined by f(x) = x ∀ x ∈A is called the identity of A and is denoted by IA



(XII) Constant function:

Afunction f :A→B is said to be a constant function if every element ofAhas the same f image in B. Thus f : A→ B ;

f(x) = c, ∀ x ∈A, c ∈ B is a constant function.



6. DOMAINS AND RANGES OF COMMON FUNCTION:

Function (y = f (x))	Domain (i.e. values taken by x)		Range (i.e. values taken by f(x))	
A. Algebraic Functions				
(i) x^n, ($n \in N$)	R = (set of real numbers)		R,	if n is odd
			$R^+ \cup \{0\}$,	if n is even
(ii) $\frac{1}{x^n}$, ($n \in N$) R – {0}			R – {0},	if n is odd
			R^+,	if n is even
(iii) $x^{1/n}$, ($n \in N$)	R,	if n is odd	R,	if n is odd
	$R^+ \cup \{0\}$,	if n is even	$R^+ \cup \{0\}$,	if n is even
(iv) $\frac{1}{x^{1/n}}$, ($n \in N$)	R – {0},	if n is odd	R – {0},	if n is odd
	R^+,	if n is even	R^+,	if n is even
B. Exponential Functions				
(i) e^x	R		R^+	
(ii) $e^{1/x}$	R – {0}		$R^+ – \{1\}$	
(iii) a^x, a > 0	R		R^+	
(iv) $a^{1/x}$, a > 0	R – {0}		$R^+ – \{1\}$	
C. Logarithmic Functions				
(i) $\log_a x$, (a > 0) ($a \neq 1$)	R^+		R	
(ii) $\log_x a = \frac{1}{\log_a x}$	$R^+ – \{1\}$		R – {0}	(a > 0) (a 1)
D. Integral Part Functions				
(i) [x]	R		I	
(ii) $\frac{1}{[x]}$	R – [0, 1)		$\left\{\frac{1}{n}, n \in I-\{0\}\right\}$	

E. Fractional Part Functions		
(i) $\{x\}$	R	[0, 1)
(ii) $\frac{1}{\{x\}}$	R – I	$(1, \infty)$
F. Modulus Functions		
(i) $\lvert x \rvert$	R	$R^+ \cup \{0\}$
(ii) $\frac{1}{\lvert x\rvert}$	R – {0}	R^+
G. Signum Function		
sgn (x) = $\frac{\lvert x\rvert}{x}, x \neq 0$	R	$\{-1, 0, 1\} = 0, x = 0$
H. Constant Function		
say f (x) = c	R	{c}

7. EQUAL OR IDENTICAL FUNCTION:

Two functions f & g are said to be equal if:

(i) The domain of f = the domain of g.

(ii) The range of f = the range of g and

(iii) f(x) = g(x), for every x belonging to their common domain. eg.

$f(x) = \frac{1}{x}$ & $g(x) = \frac{x}{x^2}$ are identical functions.

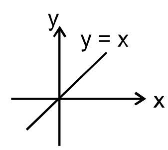
Note: Functions are also equal if their graphs are same.

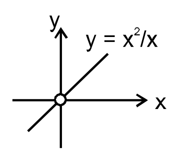
e.g. $f(x) = \frac{1}{x}$ and $g(x) = \frac{x}{x^2}$ are identical functions.

Clearly the graphs of f(x) and g(x) are exactly same

But f(x) = x and $g(x) = \frac{x^2}{x}$ are not identical functions.

Clearly the graphs of f(x) and g(x) are different at x = 0.





SOLVED EXAMPLES

Ex.10 $\log_2 x - 1 > 0$

Sol. $x - 1 > 2^0$

$x - 1 > 1$

$x > 2$

$x \in (2, \infty)$

Ex.11 The range of x for which $2 \le |x-1| \le 3$.

Sol. In this case,

$-3 \le x-1 \le -2$ or $2 \le x-1 \le 3$

$\Rightarrow -2 \le x \le -1$ or $3 \le x \le 4$

$\Rightarrow x \in [-2,-1] \cup [3,4]$

Ex.12 Find domain and range of the follow___

(i) $f(x)=x^2 + 2x + 5$ (ii) $f(x) = \cos^2 x + 1$ (iii) $f(x) = (3\sin x + 4\cos x + 5)$

Sol. (i) Polynomial function having domain all real no. & Range is $\left(\frac{-D}{4a},\infty\right)$

$D = b^2 - 4ac = 4 - 20 = -16$

Range = $(4, \infty)$

(ii) $f(x) = \cos^2 x + 1$

$D_f \in R$

R_b:- $-1 \le \cos x \le 1$

$0 \le \cos^2 x \le 1$

$1 \le \cos^2 x + 1 \le 2 = [1, 2]$

(iii) $D_f \in R$

R_b:-

$-\sqrt{a^2+b^2} \le a \sin x + b \cos x \le \sqrt{a^2+b^2}$

$-5 \le 3 \sin x + 4 \cos x \le 5$

$0 \le 3 \sin x + 4 \cos x + 5 \le 10$

$R_b \in [0, 10]$

Ex.13 Examine whether following pair of functions are identical or not?

(i) $f(x) = \frac{x^2-1}{x-1}$ and $g(x) = x + 1$

(ii) $f(x) = \sin^2 x + \cos^2 x$ and $g(x) = \sec^2 x - \tan^2 x$

Sol. (i) No, as domain of f(x) is $R - \{1\}$

while domain of g(x) is R

(ii) No, as domain are not same. Domain of f(x) is R

while that of g(x) is $R - \left\{(2n+1)\frac{\pi}{2};\ n \in I\right\}$

Ex.14 If $f : R^+ \to R^+$, $f(x) = x^2 + 2$ and

$g : R^+ \to R^+$, $g(x) = \sqrt{x+1}$

then (f + g) (x) equals -

(1) $\sqrt{x^2+3}$ (2) $x + 3$ (3) $\sqrt{x^2+2}+(x+1)$ (4) $x^2 + 2 + \sqrt{(x+1)}$

Sol. $(f + g)(x) = f(x) + g(x)$

$= x^2 + 2 + \sqrt{x+1}$

PRACTICE SECTION-02

Q.1 Find x if $\log_{0.2}(x-1) \le \log_{0.04}(x-1)$

Q.2 Find x if $\log_2 \log_{1/2} \log_3 x > 0$

Q.3 Examine whether the following pair of functions are identical or not:

(i) $f(x) = \text{sgn}(x)$ and $g(x) = \begin{cases} \frac{x}{|x|} & x \ne 0 \\ 0 & x = 0 \end{cases}$

(ii) $f(x) = \text{cosec}^2 x - \cot^2 x$ and $g(x) = 1$

ANSWER KEY

Q.1	Q.4	Q.5
$x \in [2,\infty)$	$x \in \left(1,\sqrt{3}\right)$	**(i)** Yes **(ii)** No

8. CLASSIFICATION OF FUNCTIONS:

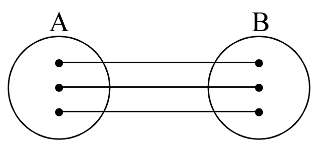
(A) One – One Function (Injective mapping):

A function f: A $\rightarrow$ B is said to be a one–one function or injective mapping if different elements of A have different

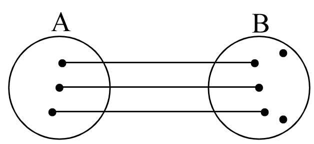
f images in B. Thus for $x_1, x_2 \in A$ & $f(x_1), f(x_2) \in B$,

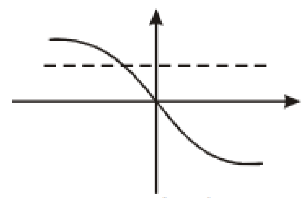
$f(x_1) = f(x_2) \Leftrightarrow x_1 = x_2$ or $x_1 \neq x_2 \Leftrightarrow f(x_1) \neq f(x_2)$.

Diagrammatically an injective mapping can be shown as



OR





one-one function

Note:

(i) Any function which is entirely increasing or decreasing in whole domain, then f(x) is one–one.

(ii) If any line parallel to x–axis cuts the graph of the function atmost at one point, then the function is one – one.

(iii) Linear function is always one-one.

(B) Many–one function:

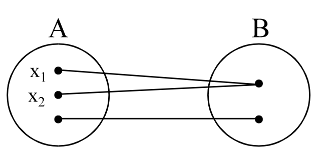
A function f: A $\rightarrow$ B is said to be a many one function if two or more elements of A have the same f image in B. hus

: A $\rightarrow$ B is many one if for; $x_1, x_2 \in A$, $f(x_1) = f(x_2)$ but $x_1 \neq x_2$.

OR

If a function is not a one-one, it will be known as many-one function.

Diagrammatically a many one mapping can be shown as



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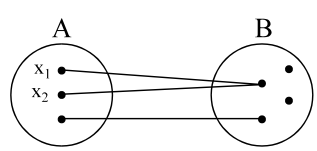
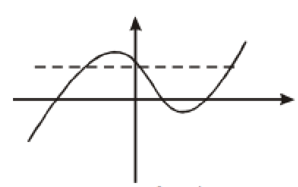




many-one function

Note:

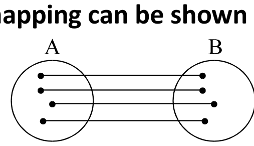
(i) Any continuous function which has atleast one local maximum or local minimum, then f(x) is many–one. In other words, if a line parallel to x–axis cuts the graph of the function atleast at two points, then f is many–one.

(ii) If a function is one–one, it cannot be many–one and vice versa.

(C) Onto function (Surjective mapping):

If the function f A $\rightarrow$ B is such that each element in B (co–domain) is the f image of atleast one element in A, then we say that f is a function of A 'onto' B. Thus f: A $\rightarrow$ B is surjective if $\forall$ b $\in$ B, $\exists$ some a $\in$ A such that f (a) = b. If range = co–domain, then f(x) is onto.

Diagrammatically surjective mapping can be shown as



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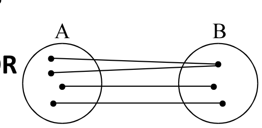


Note:

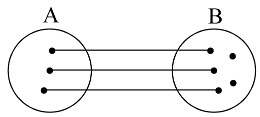
(a) Any polynomial of degree odd defined on R is onto.

(b) If co-domain of f is not given then it is taken to b

(D) Into function:

If f: A $\rightarrow$ B is such that there exists atleast one element in co–domain which is not the image of any element in domain, then f(x) is into **OR** if a function is not onto then it will be into function.

Diagrammatically into function can be shown as



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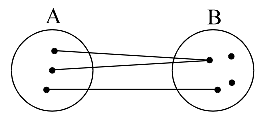


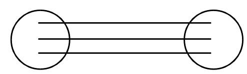
Note:

(a) If a function is onto, it cannot be into and vice versa.

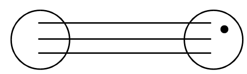
(b) A polynomial of degree even defined from R $\rightarrow$ R will always be into.

A FUNCTION CAN BE ONE OF THESE FOUR TYPES:

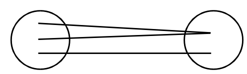
(a) one–one onto (injective & surjective)



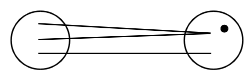
(b) one–one into (injective but not surjective)



(c) many–one onto (surjective but not injective)



(d) many–one into (neither surjective nor injective)



Note:

(i) If f is both injective & surjective, then it is called a **Bijective** mapping.
The bijective functions are also named as invertible nonsingular or biuniform functions.

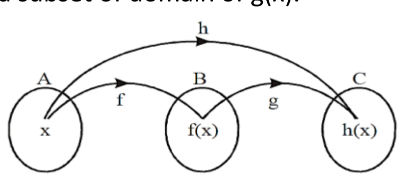
(ii) If a set A contains n distinct elements then the number of different functions defined from A $\rightarrow$ A is n^n out of it n! are one one.

9. COMPOSITE OF UNIFORMLY & NON-UNIFORMLY DEFINED FUNCTIONS:

Let f: A $\rightarrow$ B & g: B $\rightarrow$ C be two functions. Then the function gof: A $\rightarrow$ C defined by (gof) (x) = g (f(x)) $\forall$ x $\in$ A is called the composite of the two functions f & g.

Diagrammatically $\xrightarrow{x} \boxed{f} \xrightarrow{f(x)} \boxed{g}$ g (f(x)).

Thus, the image of every x $\in$ A under the function gof is the g–image of the f–image of x. Note that gof(x) exists only for those x when range of f(x) is a subset of domain of g(x).



$g\{f(x)\} = h(x)$

PROPERTIES OF COMPOSITE FUNCTIONS:

(i) The composite of functions is not commutative i.e. gof $\neq$ fog.

(ii) The composite of functions is associative i.e. if f, g, h are three functions such that fo (goh) & (fog) oh are defined, then fo (goh) = (fog) oh.

(iii) The composite of two bijections is a bijection i.e. if f & g are two bijections such that gof is defined, then gof is also a bijection.

SOLVED EXAMPLES

Ex.15 (i) Find whether $f(x) = x + \cos x$ is one-one.

(ii) Identify whether the function $f(x) = -x^3 + 3x^2 - 2x + 4$ for $f : R \to R$ is ONTO or INTO

(iii) $f(x) = x^2 - 2x + 3$; $[0, 3] \to A$. Find whether f(x) is injective or not. Also find the set A, if f(x) is surjective.

Sol. (i) The domain of f(x) is R. $f'(x) = 1 - \sin x$.

$\therefore$ $f'(x) \geq 0\ \forall\ x \in$ complete domain and equality holds at discrete points only

$\therefore$ f(x) is strictly increasing on R. Hence f(x) is one-one.

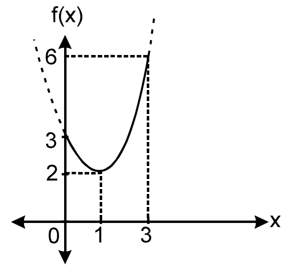
(ii) As range $\equiv$ codomain, therefore given function is ONTO

(iii) $f'(x) = 2(x - 1)$; $0 \leq x \leq 3$

$$\therefore\quad f'(x) = \begin{cases} -ve & ; \quad 0 \leq x < 1 \\ +ve & ; \quad 1 < x < 3 \end{cases}$$

$\therefore$ f(x) is non monotonic. Hence it is not injective.

For f(x) to be surjective, A should be equal to its range.



By graph range is [2, 6]

$\therefore$ $A \equiv [2, 6]$

Ex.16 Let $f(x) = \cos x + x$ and $g(x) = x^2$. Find fog(x)

Sol. fog(x) $= \cos g(x) + g(x)$

$= \cos x^2 + x^2$

Ex.17 If $f(x) = |\ |x - 3| - 2\ |$ $\quad 0 \leq x \leq 4$

$g(x) = 4 - |2 - x|$ $\quad -1 \leq x \leq 3$ then find fog (2)

Sol. fog (2) = f(4) ($\because$ g(2) = 4)

$\therefore$ fog (2) = 1

Ex.18 Let $f : R \to R$, $f(x) = \dfrac{\alpha x^2 + 6x - 8}{\alpha + 6x - 8x^2}$. Find the value of α for f(x) to be onto.

Sol. $y = \dfrac{ax^2 + 6x - 8}{\alpha + 6x - 8x^2}$

$\Rightarrow (\alpha + 8y)x^2 + 6(1-y)x - (\alpha y + 8) = 0$

According to condition, y takes all real values for real x,

i.e $D \geq 0 \forall y \in R$

$\Rightarrow 36(1-y)^2 + 4(\alpha y + 8)(\alpha + 8y) \geq 0 \forall y \in R$

$\Rightarrow (9 + 8\alpha)y^2 + (\alpha^2 + 46)y + (9 + 8\alpha) \geq 0\ \forall y \in R$

i.e. $D \leq 0$ and coefficient of $y^2 > 0$

$\Rightarrow \left(\alpha^2 + 46\right)^2 \leq 4\left(9 + 8\alpha\right)^2$ and $9 + 8\alpha > 0$

$\Rightarrow \alpha^2 - 16\alpha + 28 \leq 0$ and $\alpha > \dfrac{-9}{8}$

$\Rightarrow 2 \leq \alpha \leq 14$

Hence, $\alpha \in [2,14]$ Ans.

Ex.19 Lef f(x) = $\frac{x-1}{x+1}$, $f^2(x) = f\{f(x)\}$, $f^3(x) = f\{f^2(x)\}$ ...... $f^{k+1}(x) = f\{f^k(x)\}$, for k = 1, 2, 3, ... Find $f^{1998}(x)$

Sol. f(x) = $\frac{x-1}{x+1}$

$f^2(x) = f\{f(x)\} = \frac{f-1}{f+1} = \frac{-1}{x}$

$f^3(x) = \frac{x+1}{1-x}$

$f^4(x) = x$

$f^5(x) = f\{f^4(x)\} = f(x)$

$f^{1998}(x) = f^2(x) = \frac{-1}{x}$

Ex.20 f(x) = $\sqrt{|x-1|}$ and g (x) = sin x then (fog) (x) equals -

(1) sin $\left\{\sqrt{|x-1|}\right\}$ (2) |sin x/2 – cos x/2|

(3) | sin x – cos x | (4) None of these

Sol. (fog) (x) = f [g(x)] = f [sin x]

= $\sqrt{|\sin x-1|}$

= $\sqrt{|1-\sin x|}$

= $\sqrt{|\sin^2 x/2+\cos^2 x/2-2\sin x/2\cos x/2|}$

= $\sqrt{|(\sin x/2-\cos x/2)^2|}$

= |sin x / 2 – cos x / 2 |

Ex.21 If f : R – {3} $\to$ R – {1}, f (x) = $\frac{x-2}{x-3}$ then function f(x) is -

(1) Many one, into (2) one-one, into (3) Many one, onto (4) one-one, onto

Sol. $\because$ f (x) = $\frac{x-2}{x-3}$

$\therefore$ f ′(x) = $\frac{(x-3).1-(x-2).1}{(x-3)^2} = \frac{-1}{(x-3)^2}$

$\therefore$ f ′(x) < 0 $\forall$ x $\in$ R – {3}

$\therefore$ f (x) is monotonocally decreasing function

$\Rightarrow$ f is one-one function.

onto/ into : Let y $\in$ R – {1} (co-domain)

Then one element x $\in$ R – {3} is domain is such that

f(x) = y $\Rightarrow$ $\frac{x-2}{x-3}$ = y $\Rightarrow$ x – 2 = xy – 3y

$\Rightarrow$ x = $\left(\frac{3y-2}{y-1}\right)$ = x $\in$ R – {3}

$\therefore$ the pre-image of each element of co-domain
R – {1} exists in domain R – {3}.
$\Rightarrow$ f is onto.

Ex.22 Define fog(x) and gof(x). Also find their domain and range.

(i) $f(x) = [x]$, $g(x) = \sin x$ (ii) $f(x) = \tan x$, $x \in (-\pi/2, \pi/2)$; $g(x) = \sqrt{1-x^2}$

Sol. (i) gof = sin [x] domain: R range {sin a: a $\in$ I}

fog = [sin x] domain: R range: {–1, 0, 1}

(ii) gof $\equiv \sqrt{1-\tan^2 x}$, domain: $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ range: [0, 1]

fog $\equiv \tan\sqrt{1-x^2}$ domain: [–1, 1] range [0, tan 1]

Ex.23 Let $f(x) = e^x$: $R^+ \to R$ and $g(x) = x^2 - x$: $R \to R$. Find domain and range of fog (x) and gof (x)

fog (x)	gof (x)
Domain: $(-\infty, 0) \cup (1, \infty)$	Domain: $(0, \infty)$
Range: $(1, \infty)$	Range: $(0, \infty)$

Ex.24 Function f : N $\to$ N , f(x) = 2x + 3 is -

(1) one-one onto (2) one-one into (3) many one onto (4) many one into

Sol. f is one-one because for any $x_1, x_2 \in N$

$x_1 \neq x_2 \Rightarrow 2x_1 + 3 \neq 2x_2 + 3 \Rightarrow f(x_1) \neq f(x_2)$

Further $f^{-1}(x) = \frac{x-3}{2} \notin N$ (domain) when

x = 1, 2, 3 etc.

$\therefore$ f is into which shows that f is one- one into.

Alter

f (x) = 2x + 3

$f'(x) = 2 > 0\ \forall\ x \in N$

$\therefore$ f(x) is increasing function

$\therefore$ f(x) is one-one function

& $\because$ x = 1 , 2, 3, ...

$\therefore$ min value of f(x) is 2.1 + 3 = 5

$\therefore$ f(x) $\neq$ {1 , 2, 3, 4}

$\therefore$ Co Domain $\neq$ Range

$\therefore$ f(x) is into function

PRACTICE SECTION-03

Q.1 If f (x) = |x| and g(x) = [x], then value of

$fog\left(-\frac{1}{4}\right) + gof\left(-\frac{1}{4}\right)$ is -

(1) 0 (2) 1 (3) –1 (4) 1/4

Q.2 For each of the following functions find whether it is one-one or many-one and also into or onto

(i) $f(x) = 2\tan x$; $(\pi/2, 3\pi/2) \to R$

(ii) $\frac{1}{1+x^2}$ $f(x) = \frac{1}{1+x^2}$; $f : (-\infty, 0) \to R$

(iii) $f(x) = x^3 + x^2 + 3x + \sin x$; $R \to R$

Q.3 If $f(x) = \frac{x-3}{x+1}$, then f [f {f (x)}] equals -

(1) x (2) 1/x (3) –x (4) –1/x

Q.4 Which of the following functions defined from R to R are one-one -

(1) $f(x) = |x|$ (2) $f(x) = \cos x$ (3) $f(x) = e^x$ (4) $f(x) = x^2$

Q.5 The function $f : R \to R$, $f(x) = x^2$ is -

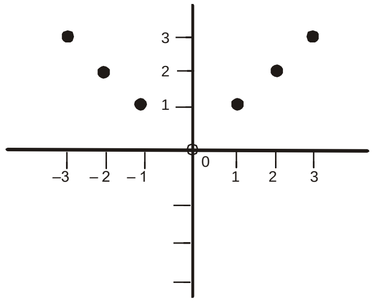
(1) one-one but not onto

(2) onto but not one-one

(3) one-one onto

(4) Many one, into

Q.6 If $f : I_0 \to N$, $f(x) = |x|$, then f is –



(1) one-one (2) onto (3) one-one onto (4) many one, into

Q.7 If $g(x) = x^2 + x - 2$ and $\frac{1}{2}$ (gof) (x) $= 2x^2 - 5x + 2$, then f(x) is equal to -

(1) $2x - 3$ (2) $2x + 3$ (3) $2x^2 + 3x + 1$ (4) $2x^2 - 3x - 1$

Q.8 Function $f : R \to R$, $f(x) = x^3 - x$ is -

(1) one-one onto (2) one-one into (3) many-one onto (4) many-one into

Q.9 If $f : R \to R$, $f(x) = 2x - 1$ and $g : R \to R$, $g(x) = x^2 + 2$, then (gof) (x) equals-

(1) $2x^2 - 1$ (2) $(2x - 1)^2$ (3) $2x^2 + 3$ (4) $4x^2 - 4x + 3$

ANSWER KEY							
Q.1	**Q.3**	**Q.4**	**Q.5**	**Q.6**	**Q.7**	**Q.8**	**Q.9**
2	1	3	4	2	1	3	4

Q.2 (i) one-one onto (ii) one-one into (iii) one-one onto

10. HOMOGENEOUS FUNCTIONS:

A function is said to be homogeneous with respect to any set of variables when each of its terms is of the same degree with respect to those variables. For example, $5x^2 + 3y^2 - xy$ is homogeneous in x & y. ymbolically if, $f(tx, ty) = t^n . f(x, y)$ then $f(x, y)$ is homogeneous function of degree n.

11. BOUNDED FUNCTION:

A function is said to be bounded if $|f(x)| \le M$, where M is a finite quantity.

12. IMPLICIT & EXPLICIT FUNCTION:

A function defined by an equation not solved for the dependent variable is called an **Implicit Function**. For eg. The equation $x^3 + y^3 = 1$ defines y as an implicit function. If y has been expressed in terms of x alone then it is called an **ExplicitFunction**.

13. ODD & EVEN FUNCTIONS:

If $f(-x) = f(x)$ for all x in the domain of 'f' then f is said to be an even function.

e.g. $f(x) = \cos x$; $g(x) = x^2 + 3$.

If $f(-x) = -f(x)$ for all x in the domain of 'f' then f is said to be an odd function.

e.g. $f(x) = \sin x$; $g(x) = x^3 + x$.

Note:

(a) $f(x) - f(-x) = 0 \Rightarrow f(x)$ is even & $f(x) + f(-x) = 0 \Rightarrow f(x)$ is odd.

(b) A function may neither be odd nor even. Ex.($f(x) = e^x$, $\cos^- x$)

(c) Inverse of an even function is not defined.

(d) Every even function is symmetric about the y–axis & every odd function is symmetric about the origin.

(e) Every function can be expressed as the sum of an even & an odd function.

e.g.

$$f(x) = \underbrace{\frac{f(x)+f(-x)}{2}}_{\text{EVEN}} + \underbrace{\frac{f(x)-f(-x)}{2}}_{\text{ODD}}$$

(f) The only function $f(x) = 0$ which is even and odd at the same time. Any non zero constant is even.

(g) If f(x) is even then f'(x) is odd while derivative of odd function is even. Note that same can not be said for integral of function.

(h)

f(x)	g(x)	f(x) + g(x)	f(x) – g(x)	$f(x)\cdot g(x)$	f(x) / g(x)	(gof)(x)	(fog)(x)
odd	odd	odd	odd	even	even	odd	odd
even	even	even	even	even	even	even	even
odd	even	neither odd nor even	neither odd nor even	odd	odd	even	even
even	odd	neither odd bor even	neither odd nor even	odd	odd	even	even

14. PERIODIC FUNCTION:

If f(x) = f(x+T) for all x in the domain of f(x), f(x) is said to be a periodic function. Smallest positive value of T is known as fundamental period of the given function. Graph of a periodic function with period T repeates itself after every interval of length T.

Properties of periodic function:

(a) If f(x) has a period T, then $\frac{1}{f(x)}$ and $\sqrt{f(x)}$ also have a period T.

(b) If f(x) has a period T, then f (ax + b) has a period $\frac{T}{|a|}$.

(c) Inverse of a periodic function does not exist.

(d) Every constant function is always periodic, with no fundamental period.

(e) If f(x) has a period T & g (x) also has a period T then it does not mean that f (x) + g (x) must have a period T. e.g. f(x) =| sin x | + | cos x |; $\sin^4 x + \cos^4 x$

(f) If f (x) and g (x) are periodic then f (x) + g (x) need not be periodic. e.g. f (x) = cos x and g (x) = { x }

(g) If period of f(x) is T_1 and g(x) is T_2 then period of f ± g, f · g, f/g is T = L.C.M. (T_1, T_2) provided there does not exist any positive real less than T after which value repeats.

Note:

(1) L.C.M. $\left(\frac{a}{b},\frac{c}{d}\right)=\frac{\text{L.C.M.}(a,c)}{\text{H.C.F.}(b,d)}$

(2) L.C.M. of rational with irrational is not possible.

15. GENERAL:

If x, y are independent variables, then :

(i) f(xy) = f(x) + f(y) $\Rightarrow$ f(x) = k *l*n x or f(x) = 0.

(ii) f(xy) = f(x). f(y) $\Rightarrow$ f(x) = x^n, n $\in$ R

(iii) f(x + y) = f(x). f(y) $\Rightarrow$ f(x) = a^{kx}.

(iv) f(x + y) = f(x) + f(y) $\Rightarrow$ f(x) = kx, where k is a constant.

SOLVED EXAMPLES

Ex.26 Show that $\log\left(x+\sqrt{x^2+1}\right)$ is an odd function.

Sol. Let f(x) = $\log\left(x+\sqrt{x^2+1}\right)$.

Then f(–x) = $\log\left(-x+\sqrt{(-x)^2+1}\right)$

$$= \log\left(\frac{\left(\sqrt{x^2+1}-x\right)\left(\sqrt{x^2+1}+x\right)}{\sqrt{x^2+1}+x}\right) = \log\frac{1}{\sqrt{x^2+1}+x} = -\log\left(x+\sqrt{x^2+1}\right) = -f(x)$$

or f(x) + f(–x) = 0

Hence f(x) is an odd function.

Ex.27 Show that $a^x + a^{-x}$ is an even function.

Sol. Let $f(x) = a^x + a^{-x}$

Then $f(-x) = a^{-x} + a^{-(-x)} = a^{-x} + a^x = f(x)$.

Hence f(x) is an even function

Ex.28 Find period of the following functions

(i) $f(x) = \{x\} + \sin x$, where {.}denotes fractional part function

(ii) $f(x) = \sin\frac{3x}{2} - \cos\frac{x}{3} - \tan\frac{2x}{3}$

(iii) $f(x) = \cos x \,.\, \cos 3x$

Sol. (i) Period of $\sin x = 2\pi$

Period of $\{x\} = 1$

but L.C.M. of 2π and 1 is not possible as their ratio is irrational number

$\therefore$ it is aperiodic

(ii) Period of f(x) is L.C.M. of $\frac{2\pi}{3/2}, \frac{2\pi}{1/3}, \frac{\pi}{2/3}$ = L.C.M. of $\frac{4\pi}{3}, 6\pi, \frac{3\pi}{2} = 12\pi$

(iii) $f(x) = \cos x \,.\, \cos 3x$

period of f(x) is L.C.M. of $\left(2\pi, \frac{2\pi}{3}\right) = 2\pi$

But 2π may or may not be fundamental periodic, but fundamental period = $\frac{2\pi}{n}$, where $n \in N$. Hence cross-checking for n = 1, 2, 3, ... we find π to be fundamental period $f(\pi + x) = (-\cos x)(-\cos 3x) = f(x)$.

Ex.29 Find the period of f(x) satisfying the condition

(i) $f(x-1) + f(x+3) = f(x+1) + f(x+5)$

(ii) $f(x) = \{x\} + \cos \pi x$

Where $\{\cdot\}$ denotes fraction part.

Sol. **(i)** $f(x-1) + f(x+3) = f(x+1) + f(x+5)$ ....(1)

Replacing x by x + 2

$f(x + 1) + f(x+5) = f(x+3) + f(x+7)$ ....(2)

Adding (1) and (2), we get

$f(x-1) = f(x+7)$ i.e. $f(x) = f(x+8)$

Hence, period is 8.

(ii) $f(x) = \{x\} + \cos \pi x$

Period of $\{x\} = 1$

Period of $\cos \pi x = \frac{2\pi}{\pi} = 2$

Hence period of f(x) = 2.

Ex.30 Consider the function $f(x) = \begin{cases} x|x|, & 0 \le x < 1 \\ 2x, & x \ge 1 \end{cases}$, Find the extension f the function $\forall x \in R$ if

(i) f(x) is even (ii) f(x) to be odd

Sol. **(i)** We have $f(x) = \begin{cases} x^2, & 0 \le x < 1 \\ 2x, & x \ge 1 \end{cases}$

If f is even $\forall x \in R$, then $f(-x) = f(x)$

Hence f(–x) =

$$\begin{cases} x^2, & -1 < x \le 0 \\ -2x, & x \le 1 \end{cases}; \qquad f(x) = \begin{cases} -2x, & x \le 1 \\ x^2, & -1 < x < 1 \\ 2x, & 1 \le x \end{cases}$$

(ii) For f(x) to be odd function

–f(x) = –x2, –1 < x $\le$ 0

= 2x, x $\le$ –1

$$f(x) = \begin{cases} 2x, & x \le -1 \\ -x^2, & -1 < x \le 0 \\ x^2, & 0 \le x < 1 \\ 2x, & 1 \le x \end{cases} \qquad \text{or } f(x) = \begin{cases} x|x|, & |x| < 1 \\ 2x, & |x| \ge 1 \end{cases}$$

Ex.31 If f(x) = cos (log x), then

f(x) f(y) – 1/2 [f (x/y) + f(xy)] is equal to

(1) –1 (2) ½ (3) –2 (4) 0

Sol. cos (log x) cos (log y)

$-\frac{1}{2}$ [cos (log x/y) + cos (log xy)]

= $\frac{1}{2}$ [cos (log x + log y) + cos (log x – log y)]

– $\frac{1}{2}$ [cos (log x – log y) + cos (log x + log y)]

= 0

PRACTICE SECTION-04

Q.1 If f(x) = $\frac{2^x + 2^{-x}}{2}$, then f (x + y) . f (x – y) is equal to -

(1) $\frac{1}{2}$ [f (x+ y) + f(x – y)]
(2) $\frac{1}{2}$ [f (2x) + f (2y)]
(3) $\frac{1}{2}$ [f(x+ y). f(x – y)]
(4) None of these

Q.2 Find the period of following function.

(i) f(x) = sin x + | sin x |
(ii) f(x) = $\sqrt{3}$ cos x – sin $\frac{x}{3}$
(iii) sin $\frac{2x}{5}$ – cos $\frac{3x}{7}$
(iv) f(x) = $\sin^4 x + \cos^4 x$

Q.3 Determine whether the following functions are even or odd?

(i) $\frac{e^x + e^{-x}}{e^x - e^{-x}}$ (ii) $\sin(x^2) + x(\tan x)$ (iii) $x^3 \ell n\left(\frac{1+x}{1-x}\right)$

Q.4 The period of sin (x/3), is

[1] 2π [2*] 6π [3] $\pi/2$ [4] $\pi/3$

Q.5 The period of | cos x |, is

[1] 2π [2*] π [3] $\pi/2$ [4] $3\pi/2$

Q.6 The period of $x - [x]$, is, where [x] is greatest integer function.

[1] π [2] 2 [3*] 1 [4] None

Q.7 Which of the following is a period function

(1) $\sin x^2$ (2) $x + \sin x$ (3) $[x]$ (4) None

Q.8 The period of the function $f(x) = \log \cos 2x + \tan 4x$, is

(1) $\pi/2$ (2) π (3) 2π (4) $2\pi/5$

Q.9 The period of $f(x) = \tan(3x + 2)$, is

(1) π (2) $2\pi/3$ (3) $\pi/3$ (4) None

Q.10 The period of $f(x) = |\sin x| + |\cos x|$, is

(1) $\pi/2$ (2) π (3) 2π (4) None

Q.11 If $f(x)$ is an odd function differentiable on R, then $f'(x)$ is an function

(1) even (2) odd (3) neithe even nor odd (4) None

Q.12 If $f(x)$ is an even function, then the curve $y = f(x)$ is symmetric about

(1) x-axis (2) y - axis (3) both the axes (4) None

Q.13 If $f(x)$ is an odd, function, then the curve $y = f(x)$ is symmetric about

(1) x-axis (2) y - axis (3) both the axes (4) In opposite quadrant

Q.14 $f(x) = \log\left(x + \sqrt{1 + x^2}\right)$, is

(1) odd (2) even (3) neither even nor odd (4) None

ANSWER KEY

Q.1	Q.4	Q.5	Q.6	Q.7	Q.8	Q.9	Q.10	Q.11	Q.12	Q.13	Q.14
2	2	2	3	4	2	3	2	1	2	4	1

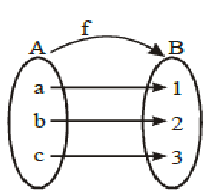
Q.2 (i) 2π (ii) 6π (iii) 70π (iv) $\frac{\pi}{2}$

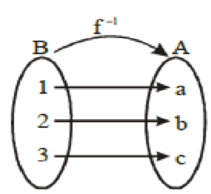
Q.3 (i) Odd (ii) Even (iii) Even

16. INVERSE OF A FUNCTION:

Let $f : A \to B$ be a one–one & onto function, then their exists a unique function

$g : B \to A$ such that $f(x) = y \Leftrightarrow g(y) = x$, $\forall\, x \in A$ & $y \in B$. Then g is said to be inverse of f.





Note:

(a) f(x) and g(x) are said to be inverse of each other. g(x) is also denoted by f^{-1} (x) and f(x) is denoted by g^{-1} (x).

(b) Domain of f(x) = Range of g(x)

(c) Range of f(x) = Domain of g(x)

Properties of Inverse Function:

(i) The inverse of a bijection is unique.

(ii) If $f : A \to B$ is a bijection & $g : B \to A$ is the inverse of f, then fog = I_B and gof = I_A , where I_A & I_B are identity functions on the sets A & B respectively.

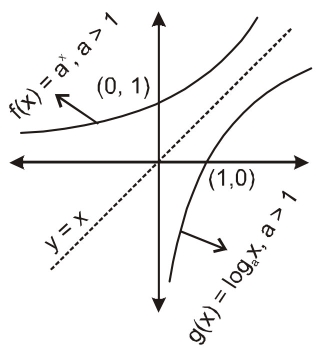
(iii) The inverse of a bijection is also a bijection.

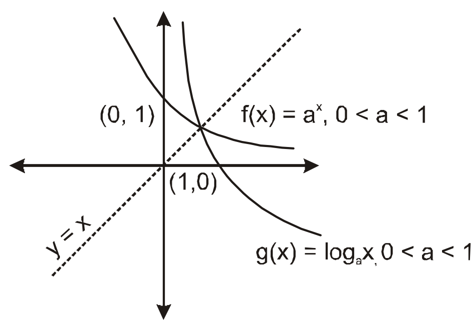
(iv) If $f : A \to B$ and $g : B \to C$ are two bijections, then gof : $A \to C$ is a bijection and $(gof)^{-1} = f^{-1}og^{-1}$

(v) fog $\neq$ gof but if fog = gof then either $f^{-1} = g$ or $g^{-1} = f$ also (fog) (x) = (gof) (x) = x

(vi) The graphs of f and g are the mirror images of each other in the line y = x.

For Ex.f(x) = a^x and g(x) = $\log_a x$ are inverse of each other, and their graphs are mirror images of each other on the line y = x as shown below.

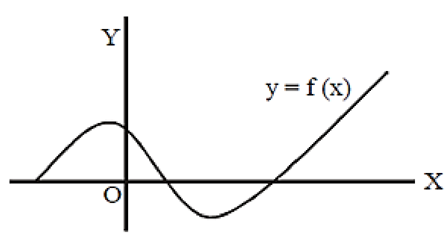




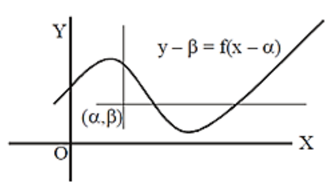
(vii) If f(x) and g(x) are inverse function of each other, then $f'(g(x)) = \dfrac{1}{g'(x)}$

17. SOME GRAPHICAL TRANSFORMATIONS:

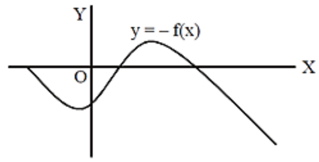
Consider the graph y= f(x) shown alongside.



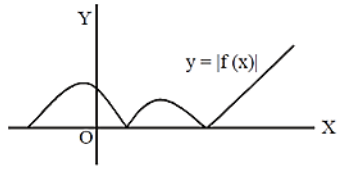
(i) Graph of $y - \beta = f(x - \alpha)$ is drawn by shifting the origin to (α, β) & then translating the graph of y= f(x) w.r.t. new axes



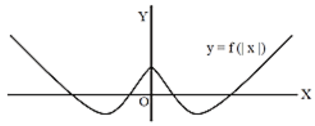
(ii) The graph of y= – f(x) is the mirror image of f(x) inX-axis.



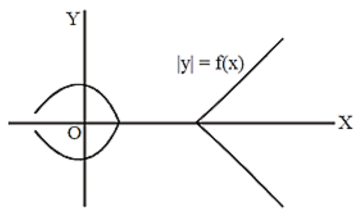
(iii) y= |f (x)| ismirror image of negative portion of y= f(x) in X-axis.



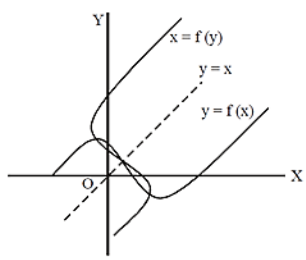
(iv) y= f (| x |) is drawn by taking the mirror image of positive x-axis graph in y-axis.



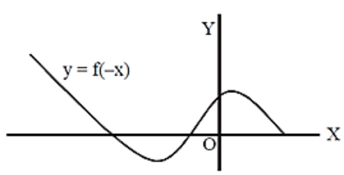
(v) The graph of |y| = f(x) is drawn by deleting those portions of the graph y= f(x) which lie below the X-axis and then taking the mirror image of the remaining portion in the X-axis, as shown along side.



(vi) x = f(y) is drawn bytakingmirror image of y= f(x) in the line y= x.



(vii) y= f(–x) is drawn bytaking themirror image of y= f(x) inY-axis.



SOLVED EXAMPLES

Ex.32 **(i)** Determine whether f(x) = $\frac{2x+3}{4}$ for f : R $\rightarrow$ R, is bijective or not ? If so find it $f^{-1}(x)$

(ii) Let f(x) = $x^2 + 2x$; $x \geq -1$. Draw graph of $f^{-1}(x)$ also find the number of Solutions of the equation, f(x) =$f^{-1}(x)$

(iii) If y=f(x)=$x^2 - 3x + 1$, $x \geq 2$. Find the value of $g'(1)$ where g is inverse of f

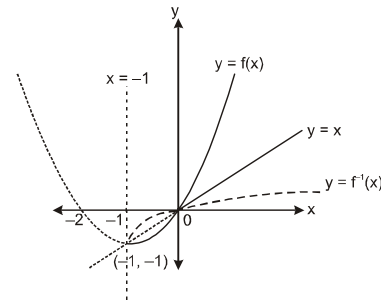
Sol. **(i)** Given function is one-one and onto, therefore it is invertible.

$$y = \frac{2x+3}{4} \Rightarrow x = \frac{4y-3}{2} \therefore f^{-1}(x) = \frac{4x-3}{2}$$

(ii) $f(x) = f^{-1}(x)$ is equivalent to $f(x) = x$

$\Rightarrow x^2 + 2x = x \Rightarrow x(x + 1) = 0 \Rightarrow x = 0, -1$

Hence two Sol. for $f(x) = f^{-1}(x)$



(iii) $y = 1 \Rightarrow x^2 - 3x + 1 = 1$

$\Rightarrow x(x-3) = 0 \Rightarrow x = 0, 3$

But $x \geq 2 \therefore x = 3$

Now $g(f(x)) = x$

Differentiating both sides w.r.t. x

$$\Rightarrow g'(f(x)).\ f'(x) = 1 \Rightarrow g'(f(x)) = \frac{1}{f'(x)}$$

$$\Rightarrow g'(f(3)) = \frac{1}{f'(3)} \Rightarrow g'(1) = \frac{1}{6-3} = \frac{1}{3} = \text{ (As } f'(x) = 2x-3)$$

Alternate Method

$y = x^2 - 3x + 1$

$x^2 - 3x + 1 - y = 0$

$$x = \frac{3 \pm \sqrt{9-4(1-y)}}{2} = \frac{3 \pm \sqrt{5+4y}}{2}$$

$x \geq 2$

$$x = \frac{3+\sqrt{5+4y}}{2}$$

$$g(x) = \frac{3+\sqrt{5+4x}}{2}$$

$$g'(x) = 0 + \frac{1}{4\sqrt{5+4x}}4$$

$$g'(1) = \frac{1}{\sqrt{5+4}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$$

Ex.33 Find the inverse of the function $f(x) = \log_a\left(x+\sqrt{x^2+1}\right), a > 0, a \neq 1$.

Sol. Hence $\sqrt{x^2+1} > 0\ \forall\ x \in R$

f(x) is one-one onto hence invertible

$y = \log_a\left(x+\sqrt{x^2+1}\right)$

$a^y = x + \sqrt{x^2+1}$ ....(i)

$a^{-y} = \dfrac{1}{x+\sqrt{x^2+1}} = -x+\sqrt{x^2+1}$ ...(ii)

(i)–(ii)

$a^y - a^{-y} = 2x \qquad \Rightarrow x = \dfrac{1}{2}\left(a^y - a^{-y}\right)$

Hence $f^{-1}(x) = \dfrac{1}{2}\left(a^x - a^{-x}\right)$

Ex.34 The inverse of the function $y = [1 - (x-3)^4]^{1/7}$ is

(1) $3 + (1 - x^7)^{1/4}$ (2) $3 - (1 - x^7)^{1/4}$ (3) $3 - (1 + x^7)^{1/4}$ (4) None of these

Sol. Clearly y is one-one and onto

we have, $y = [1 - (x-3)^4]^{1/7} \Rightarrow (x-3)^4 = 1 - y^7$

$\Rightarrow x = 3 + (1 - y^7)^{1/4}$

i.e. $f^{-1}(y) = 3 + (1 - y^7)^{1/4}$

Ex.35 Suppose $f(x) = (x+1)^2$ for $x \geq -1$. If $g(x)$ is the function whose graph is the reflection of the graph of $f(x)$ with respect to the line $y = x$, then $g(x)$ equals

[1] $\sqrt{x} - 1, x \geq 0$ [2] $\dfrac{1}{(x+1)^2}, x > -1$ [3] $\sqrt{x+1}, x \geq -1$ [4] $\sqrt{x-1}, x \geq 0$

Sol. The graph of $g(x)$ in the reflection of $f(x)$ w.r.t. the line $y = x$.

$\therefore$ $g(x)$ is the inverse of $f(x)$.

Let $y = f(x) = (x+1)^2, x \geq -1$

$\Rightarrow x + 1 = \sqrt{y} \qquad \Rightarrow x = \sqrt{y} - 1$

$\therefore f^{-1}(x) = \sqrt{x} - 1$

Also $x \geq -1 \qquad \Rightarrow f(x) \geq 0 \qquad \Rightarrow R(f) = [0, \infty)$

$\Rightarrow D(f^{-1}) = [0, \infty)$

$\therefore f^{-1}(x) = \sqrt{x} - 1$ for $x \geq 0$.

PRACTICE SECTION-05

Q.1 Determine $f^{-1}(x)$, if given function is invertible

$f : (-\infty, -1) \to (-\infty, -2)$ defined by $f(x) = -(x+1)^2 - 2$

Q.2 If $f : R \to R$, $f(x) = 4x^3 + 3$, then $f^{-1}(x)$ equals-

(1) $\left(\dfrac{x-3}{4}\right)^{1/3}$ (2) $\left(\dfrac{x^{1/3}-3}{4}\right)$ (3) $\dfrac{1}{4}(x-3)^{1/3}$ (4) None of these

Q.3 If $f : R \to R$, $f(x) = 2x + 1$ and $g : R \to R$, $g(x) = x^3$, then $(gof)^{-1}(27)$ equals -

(1) –1 (2) 0 (3) 1 (4) 2

Q.4 If $f : R \to R$ is an invertible function such that f(x) and $f^{-1}(x)$ are also mirror image to each other about the line y = –x, then

(1) f(x) is odd

(2) f(x) and $f^{-1}(x)$ may not be mirror image about the line y = x

(3) f(x) may not be odd

(4) f(x) is even

Q.5 If f(x) = $\frac{ax+b}{cx+d}$, then (fof) (x) = x, provided that

(1) d + a = 0 (2) d – a = 0 (3) a = b = c = d = 1 (4) a = b = 1

Q.6 Let f(x) = $\begin{cases} x & -1 \le x \le 1 \\ x^2 & 1 < x \le 2 \end{cases}$ the range of $h^{-1}(x)$, where h(x) = fof(x) is

(1) $\left[-1, \sqrt{2}\right]$ (2) [–1, 2] (3) [–1, 4] (4) [–2, 2]

ANSWER KEY				
Q.2	**Q.3**	**Q.4**	**Q.5**	**Q.6**
1	3	1	1	1

Q.1 $-1-\sqrt{-x-2}$

EXERSICE-I TOPIC-WISE QUESTIONS

DEFINITION OF FUNCTION

Q.1 Which of the following relation is a function ?

(1) {(1,4), (2,6), (1,5), (3,9)}

(2) {(3,3), (2,1), (1,2), (2,3)}

(3) {(1,2), (2,2,), (3,2), (4,2)}

(4) {(3,1), (3,2), (3,3), (3,4)}

Q.2 If x, y $\in$ R, then which of the following rules is not a function-

(1) $y = 9 - x^2$ (2) $y = 2x^2$

(3) $y = \sqrt{x} - |x|$ (4) $y = x^2 + 1$

Q.3 Which of the following is not a functions ?

(1) $\{(x, y) \mid x, y \in R, x^2 = y\}$

(2) $\{(x, y) \mid x, y \in R, y^2 = x\}$

(3) $\{(x, y) \mid x, y \in R, x = y^3\}$

(4) $\{(x, y) \mid x, y \in R, y = x^3\}$

Q.4 Which of the following statement given below is different from other

(1) $f : A \to B$

(2) $f : x \to f(x)$

(3) f is mapping from f to B

(4) f is a function from A to B

DOMAIN, CO-DOMAIN AND RANGE OF FUNCTION

Q.5 Domain of the function log $|x^2 - 9|$ is-

(1) R (2) R– [–3, 3]

(3) R – {–3, 3} (4) None of these

Q.6 The domain of the function f(x) = $\dfrac{\sqrt{-\log_{0.3}(x-1)}}{\sqrt{x^2+2x+8}}$ is

(1) (1, 4) (2) (– 2, 4)

(3) (2, 4) (4) $[2, \infty)$

Q.7 Range of f(x) = $\ell n\,(3x^2 - 4x + 5)$ is

(1) $\left[\ell n\dfrac{11}{3}, \infty\right)$ (2) $[\ell n\, 10, \infty)$

(3) $\left[\ell n\dfrac{11}{6}, \infty\right)$ (4) $\left[\ell n\dfrac{11}{12}, \infty\right)$

Q.8 Range of f(x) = $4^x + 2^x + 1$ is

(1) $(0, \infty)$ (2) $(1, \infty)$

(3) $(2, \infty)$ (4) $(3, \infty)$

Q.9 Range of f(x) = $\log_{\sqrt{5}}(\sqrt{2}(\sin x - \cos x) + 3)$ is

(1) [0, 1] (2) [0, 2]

(3) $\left[0, \dfrac{3}{2}\right]$ (4) [1, 2]

Q.10 Domain of definition of the function f(x) = $\dfrac{3}{4-x^2} + \log_{10}(x^3 - x)$, is :

(1) (1,2) (2) $(-1,0) \cup (1,2)$

(3) $(1,2) \cup (2, \infty)$ (4) $(-1,0) \cup (1,2) \cup (2, \infty)$

Q.11 Range of the function f(x) = $\dfrac{(x-2)^2}{(x-1)(x-3)}$ is

(1) $(1, \infty)$ (2) $(-\infty, 1)$

(3) R – (0, 1] (4) (0, 1]

Q.12 Range of the function f(x) = $\dfrac{x-2}{x^2-4x+3}$ is

(1) $(-\infty, 0)$ (2) R

(3) $(0, \infty)$ (4) R – {0}

Q.13 Domain of the function $\dfrac{x}{x^2-3x+2}$ is

(1) R (2) R – (1, 2)

(3) R – [1, 2] (4) R – {1, 2}

Q.14 Range of the function $f(x) = \dfrac{1}{2-\cos 3x}$ is

(1) $\left[\dfrac{1}{3}, 1\right]$ (2) $\left[0, \dfrac{1}{3}\right]$

(3) $\left(\dfrac{1}{3}, 1\right)$ (4) None of these

Q.15 Range of the function $f(x) = \dfrac{x}{1+x^2}$ is

(1) [–2, 2] (2) (2, –2)

(3) $\left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$ (4) $\left(-\dfrac{1}{2}, \dfrac{1}{2}\right)$

Q.16 The domain of the function-
f (x) = $\sqrt{x-1} + \sqrt{6-x}$ is-

(1) (1, 6) (2) [1, 6]

(3) $[1, \infty)$ (4) $(-\infty, 6]$

Q.17 The domain of the function

$f(x) = \sqrt{(2-2x-x^2)}$ is -

(1) $-\sqrt{3} \le x \le \sqrt{3}$ (2) $-1-\sqrt{3} \le x \le -1+\sqrt{3}$

(3) $-2 \le x \le 2$ (4) $-2+\sqrt{3} \le x \le -2-\sqrt{3}$

Q.18 The range of the function f : R→R, $f(x) = \tan^{-1} x$ is-

(1) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (2) $\left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$

(3) R (4) None of these

Q.19 The range of $f(x) = \sin \frac{\pi}{2} [x]$ is -

(1) {–1, 1} (2) {–1, 0, 1}

(3) [–1, 1] (4) {0, 1}

Q.20 Domain and range of $f(x) = \frac{|x-3|}{x-3}$ are respectively-

(1) R, [–1, 1] (2) R– {3}, {1, –1}

(3) R^+, R (4) None of these

Q.21 Domain of the function $f(x) = \frac{1}{\sqrt{x+2}}$ is-

(1) R (2) $(-2, \infty)$

(3) $[2, \infty]$ (4) $[0, \infty]$

Q.22 The domain where function $f(x) = 2x^2 - 1$ and $g(x) = 1 - 3x$ are equal, is-

(1) {1/2} (2) {2}

(3) {1/2, 2} (4) {1/2, –2}

Q.23 The domain of the function $\log 9 - x^2$ is-

(1) (–3, 3) (2) $(-\infty, 3)$

(3) (0, 3) (4) $(3, \infty)$

Q.24 The domain of the function f(x) = sin 1/x is -

(1) R (2) R^+ (3) R_0 (4) R^-

Q.25 Range of the function f(x) = 9 – 7 sin x is-

(1) (2, 16] (2) [2, 16]

(3) [–1, 1] (4) (2, 16)

Q.26 If the domain of the function $f(x) = \frac{|x|}{x}$ be [3, 7] then its range is-

(1) [–1, 1] (2) {–1, 1}

(3) {1} (4) {–1}

Q.27 The domain of the function $f(x) = \frac{1}{\sqrt{x-[x]}}$ is-

(1) R (2) R–Z

(3) Z (4) None of these

Q.28 For real values of x, range of function

$y = \frac{1}{2-\sin 3x}$ is -

(1) $\frac{1}{3} \le y \le 1$ (2) $-\frac{1}{3} \le y \le 1$

(3) $-\frac{1}{3} > y > -1$ (4) $\frac{1}{3} > y > 1$

Q.29 If f : R→R, $f(x) = x^2$, then {x|f(x) = –1} equals-

(1) {1} (2) {–1, 1}

(3) ϕ (4) None of these

Q.30 The range of f(x) = cos 2x – sin 2x contains the set -

(1) [2, 4] (2) [–1, 1] (3) [–2, 2] (4) [–4, 4]

ALGEBRA OF FUNCTIONS

Q.31 A function f : R → R satisfies the condition, $x^2 f(x) + f(1-x) = 2x - x^4$. Then f(x) is

(1) $-x^2 - 1$ (2) $-x^2 + 1$

(3) $x^2 - 1$ (4) $-x^4 + 1$

Q.32 Let f(x) = |x – 1|. Then :

(1) $f(x^2) = (f(x))^2$

(2) f(x + y) = f(x) + f(y)

(3) f(|x|) = |f(x)|

(4) f(1 + x) is even

EQUAL OR IDENTICAL FUNCTIONS

Q.33 Which of the following pair of functions are identical –

(1) $f(x) = \sin^2 x + \cos^2 x$ and g(x) = 1

(2) $f(x) = \sec^2 x - \tan^2 x$ and g(x) = 1

(3) $f(x) = \operatorname{cosec}^2 x - \cot^2 x$ and g(x) = 1

(4) $f(x) = \ell n x^2$ and $g(x) = 2\ell n x$

Q.34 If f (x) = 2x and g(x) is identity function, then

(1) (fog) (x) = g (x)

(2) (g + g) (x) = g (x)

(3) (fog) (x) = (g + g) (x)

(4) None of these

CLASSIFICATION OF FUNCTIONS

Q.35 If $f : R_0 \to R_0$, $f(x) = \frac{1}{x}$, then f is -
(1) onto but not one-one
(2) one-one but not onto
(3) neither one-one nor onto
(4) both one-one and onto

Q.36 Function $f : R \to R$, $f(x) = x + |x|$ is
(1) one-one (2) onto
(3) one-one onto (4) None of these

Q.37 Function $f : \left[\frac{\pi}{2}, \frac{3\pi}{2}\right] \to R$, $f(x) = \tan x$ is
(1) one-one (2) onto
(3) one-one onto (4) None of these

Q.38 If $f : I \to I, f(x) = x^3 + 1$, then f is -
(1) one-one but not onto
(2) onto but not one-one
(3) One-one onto
(4) None of these

Q.39 Function $f : R \to R$, $f(x) = x\,|x|$ is -
(1) one-one but not onto
(2) onto but not one-one
(3) one-one onto
(4) neither one-one nor onto

Q.40 Function $f : \left[\frac{\pi}{2}, \frac{3\pi}{2}\right] \to [-1,1]$, $f(x) = \sin x$ is -
(1) one-one (2) onto
(3) one-one onto (4) None of these

Q.41 $f : N \to N$ where $f(x) = x - (-1)^x$ then f ' is -
(1) one-one and into (2) many-one and into
(3) one-one and onto (4) many-one and onto

Q.42 Which of the following functions from Z to itself are bijections ?
(1) $f(x) = x^2 + x$ (2) $f(x) = x + 2$
(3) $f(x) = 2x + 1$ (4) $f(x) = x^3$

Q.43 Which of the following function is onto ?
(1) $f : R \to R$; $f(x) = 3^x$
(2) $f : R \to R^+$; $f(x) = e^{-x}$
(3) $f: [0,\pi/2] \to [-1,1]$; $f(x) = \sin x$
(4) $f : R \to R$: $f(x) = \cosh x$

Q.44 Which of the following function defined from R to R is onto ?
(1) $f(x) = e^{-x}$ (2) $f(x) = |x|$
(3) $f(x) = x^3$ (4) $f(x) = \sin x$.

Q.45 If $f : R\to R$, $f(x) = e^x + e^{-x}$, then f is -
(1) one-one but not onto
(2) onto but not one-one
(3) neither one-one nor onto
(4) both one-one and onto

Q.46 If $f : R\to R$, $f(x) = \sin^2 x + \cos^2 x$, then f is -
(1) one-one but not onto
(2) onto but not one-one
(3) neither one-one nor onto
(4) both one-one onto

Q.47 Let $f : R \to R$ be a function defined by $f(x) = \frac{2x^2 - x + 5}{7x^2 + 2x + 10}$, then f is :
(1) one – one but not onto
(2) onto but not one – one
(3) onto as well as one – one
(4) neither onto nor one – one

Q.48 Let $f : R \to R$ be a function defined by $f(x) = x^3 + x^2 + 3x + \sin x$. Then f is:
(1) one – one and onto (2) one – one and into
(3) many one and onto (4) many one and into

Q.49 If $f : [0, \infty) \to [0, \infty)$, and $f(x) = \frac{x}{1 + x}$, then f is:
(1) one-one and onto
(2) one-one but not onto
(3) onto but not one-one
(4) neither one-one nor onto

Q.50 If the functions f(x) and g(x) are defined on $R\to R$ such that $f(x) = \begin{cases} 0 & , \ x \in \text{rational} \\ x & , \ x \in \text{irrational} \end{cases}$,
$g(x) = \begin{cases} 0 & , \ x \in \text{irrational} \\ x & , \ x \in \text{rational} \end{cases}$, then $(f - g)(x)$ is
(1) one-one and onto
(2) neither one-one nor onto
(3) one-one but not onto
(4) onto but not one-one

COMPOSITE FUNCTION

Q.51 If $f : R \to R$, $f(x) = x^2 + 2x - 3$ and $g : R \to R$, $g(x) = 3x - 4$, then the value of fog (x) is-
(1) $3x^2 + 6x - 13$
(2) $9x^2 - 18x + 5$
(3) $(3x - 4)^2 + 2x - 3$
(4) None of these

Q.52 If $f : R \to R$, $f(x) = x^2 - 5x + 4$ and $g : R \to R$, $g(x) = \log x$, then the value of (gof) (2) is -
(1) 0 (2) $-\infty$
(3) ∞ (4) Undefined

Q.53 If $f : R \to R$, $g : R \to R$ and $f(x) = 3x + 4$ and (gof) (x) = $2x - 1$, then the value of g(x) is-
(1) $2x - 1$ (2) $2x - 11$
(3) $\frac{1}{3}(2x - 11)$ (4) None of these

Q.54 If f(x) = 2x and g is identity function, then-
(1) (g + g) (x) = g(x)
(2) (fog) (x) = g(x)
(3) (fog) (x) = (g + g) (x)
(4) None of these

Q.55 gof exists, when-
(1) domain of f = domain of g
(2) co-domain of f = domain of g
(3) co-domain of g = domain of g
(4) co-domain of g = co-domain of f

Q.56 If $f : R \to R$, $g : R \to R$ and $g(x) = x + 3$ and (fog) (x) = $(x + 3)^2$, then the value of f(–3) is -
(1) 0 (2) –9
(3) 9 (4) None of these

Q.57 If f(x) = ax + b and g(x) = cx + d, then f(g(x)) = g(f(x)) is equivalent to-
(1) f(1) = g(3) (2) f(2) = g(2)
(3) f(4) = g(2) (4) f(3) = g(1)

Q.58 If $f:[0,1] \to [0,1]$, $f(x) = \frac{1-x}{1+x}$. $g : [0,1] \to [0,1]$, $g(x) = 4x(1-x)$, then (fog) (x) equals-
(1) $\frac{1-4x+4x^2}{1+4x-4x^2}$ (2) $\frac{8x(1-x)}{(1+x)^2}$
(3) $\frac{1-4x-4x^2}{1+4x-4x^2}$ (4) None of these

Q.59 If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ and $g(x) = \left(\frac{3x+x^3}{1+3x^2}\right)$, then f[g(x)] is equal to-
(1) $[f(x)]^3$ (2) 3f(x)
(3) –f(x) (4) None of these

Q.60 If $f(y) = \frac{y}{\sqrt{1-y^2}}$, $g(y) = \frac{y}{\sqrt{1+y^2}}$, then (fog)(y) equals -
(1) $\frac{y}{\sqrt{1-y^2}}$ (2) $\frac{y}{\sqrt{1+y^2}}$
(3) y (4) $\frac{1-y^2}{1+y^2}$

Q.61 If $f(x) = \frac{1-x}{1+x}$, then f [f (sinθ)] equals -
(1) $\sin\theta$ (2) $\tan(\theta/2)$
(3) $\cot(\theta/2)$ (4) $\operatorname{cosec}\theta$

Q.62 If $f(x) = (a - x^n)^{1/n}$, $n \in N$, then f [f(x)] =
(1) x^n (2) x
(3) 0 (4) $(a^n - x)^n$

IMPLICIT, EXPLICIT FUNCTIONS

Q.63 Which of the following is implicit functions –
(1) $y = x^3 + 4x^2 + 5x$
(2) x + y = 1
(3) y = 1– x
(4) y = x + 1

EVEN AND ODD FUNCTION

Q.64 Which of the following is an even function?
(1) $x\frac{a^x-1}{a^x+1}$ (2) tan x
(3) $\frac{a^x-a^{-x}}{2}$ (4) $\frac{a^x+1}{a^x-1}$

Q.65 In the following, odd function is -
(1) $x^2 - |x|$ (2) $(e^x + 1)/(e^x - 1)$
(3) $\cos x^2$ (4) None of these

Q.66 The function $f(x) = x^2 - |x|$ is-
(1) an odd function (2) a rational function
(3) an even function (4) None of these

Q.67 Which one of the following is not an odd function-
(1) sin x (2) tan x
(3) cos x (4) None of these

Q.68 The function f(x) = $\frac{|\sin x|+|\cos x|}{x+\sin x}$ is -
(1) odd (2) Even
(3) odd and periodic (4) neither even nor odd

Q.69 f (x) = log (x + $\sqrt{1+x^2}$) is
(1) even function
(2) odd function
(3) neither even nor odd
(4) constant

Q.70 A function whose graph is symmetrical about the y-axis is given by-
(1) f(x + y) = f(x) + f(y) for all x, y $\in$ R
(2) f(x) = $\log_e (x + \sqrt{x^2+1})$
(3) f(x) = cos x + sin x
(4) None of these

Q.71 The function f(x) = log $\left(\frac{1+\sin x}{1-\sin x}\right)$ is
(1) even
(2) odd
(3) neither even nor odd
(4) both even and odd

Q.72 The function f(x) = [x] + $\frac{1}{2}$, x $\notin$ I is a/an (where [.] denotes greatest integer function)
(1) Even
(2) odd
(3) neither even nor odd
(4) Even as well as odd

PERIODIC FUNCTION

Q.73 The period of $\sin^8 x + \cos^8 x$ is -
(1) π (2) $\pi/2$
(3) 2π (4) None of these

Q.74 The period of function sin $\left(\frac{\pi x}{4}\right)$ + cos $\left(\frac{\pi x}{4}\right)$ is-
(1) 4 (2) 6 (3) 12 (4) 24

Q.75 The period of the function
f(x) = log cos 2x + tan 4x is-
(1)$2\pi/5$ (2) π (3) 2π (4) $\pi/2$

Q.76 The period of function |sin 2x| is -
(1) π (2) $\pi/2$ (3) 4π (4) 2π

Q.77 In the following which function is not periodic-
(1) $\cos^2 x$ (2) cos $2\pi x$ (3) cos x^2 (4) tan 4x

Q.78 The graph of the function y = f(x) is symmetrical about the line x = 2, then :
(1) f(x + 2) = f(x – 2) (2) f(2 + x) = f(2 – x)
(3) f(x) = f(–x) (4) f(x) = –f(–x)

Q.79 Fundamental period of f(x) = sec (sin x) is
(1) $\frac{\pi}{2}$ (2) 2π
(3) π (4) aperiodic

Q.80 If f (x) = sin $\left(\sqrt{[a]}\, x\right)$ (where [.] denotes the greatest integer function) has π as its fundamental period, then
(1) a = 1 (2) a = 9
(3) a $\in$ [1, 2) (4) a $\in$ [4, 5)

Q.81 The period of the function f (x) = cos $\left(\frac{8x+5}{4\pi}\right)$ is
(1) 2π (2) π
(3) π^2 (4) None of these

Q.82 The period of the function f (x) = 7cos($3x$ + 5) is
(1) 2π (2) $\frac{2\pi}{3}$
(3) $\frac{\pi}{3}$ (4) None of these

FUNCTIONAL EQUATIONS

Q.83 If $f(x) = a(x^n + 3)$; $f(1) = 12$, $f(3) = 36$; then $f(2)$ is equal to
(1) 18 (2) 24 (3) 21 (4) 27

Q.84 If f (x) is a polynomial satisfying $f(x).f(1/x)=f(x)+f(1/x)$ and $f(3) = 28$, then $f(4)$ is given by -
(1) 63 (2) 65 (3) 67 (4) 68

Q.85 If f(x) = cos (log x), then $\frac{f(xy)+f(x/y)}{f(x)f(y)}$ equals-
(1) 0 (2) –1 (3) 1 (4) 2

Q.86 If f is a real function satisfying the relation f(x + y) = f (x) f(y) for all x, y $\in$ R and f(1) = 2, then a $\in$ N, for which $\sum_{k=1}^{n} f(a+k)$ = $16(2^n - 1)$, is given by -
(1) 2 (2) 4
(3) 3 (4) None of these

INVERSE FUNCTION

Q.87 Which of the following functions has its inverse-

(1) $f : R_0 \to R^+$, $f(x) = |x|$

(2) $f : R \to R$, $f(x) = |x| + |x - 1|$

(3) $f : R \to R$, $f(x) = a^x$

(4) $f : [\pi, 2\pi] \to [-1,1]$, $f(x) = \cos x$

Q.88 If $f : R \to R$, $f(x) = x^2 + 3$, then pre-image of 2 under f is –

(1) {1,–1} (2) {–1} (3) {1} (4) ϕ

Q.89 If $f : [1, \infty) \to [2, \infty)$ is given by $f(x) = x + \frac{1}{x}$ then $f^{-1}(x)$ equals -

(1) $\frac{x+\sqrt{x^2-4}}{2}$ (2) $\frac{x}{1+x^2}$

(3) $\frac{x-\sqrt{x^2-4}}{2}$ (4) $1 + \sqrt{x^2-4}$

Q.90 If $f(x) = \log_e(x + \sqrt{1+x^2})$, then $f^{-1}(x)$ equals-

(1) $\log(x - \sqrt{1+x^2})$ (2) $\frac{e^x+e^{-x}}{2}$

(3) $\frac{e^x-e^{-x}}{2}$ (4) $\frac{e^x-e^{-x}}{e^x+e^{-x}}$

Q.91 If $f(x) = x^3 - 1$ and domain of f = {0, 1, 2, 3}, then domain of f^{-1} is -

(1) {0, 1, 2, 3}

(2) {1, 0, –7, –26}

(3) {–1, 0, 7, 26}

(4) {0, –1, –2, –3}

Q.92 If function $f : R \to R^+$, $f(x) = 2^x$, then $f^{-1}(x)$ will be equal to-

(1) $\log_2(1/x)$ (2) $\log_x 2$

(3) $\log_2 x$ (4) None of these

Q.93 The inverse of the function $f(x) = \frac{e^x-e^{-x}}{e^x+e^{-x}} + 2$ is given by -

(1) $\log\left(\frac{x-2}{x-1}\right)^{1/2}$ (2) $\log\left(\frac{x-1}{x+1}\right)^{1/2}$

(3) $\log\left(\frac{x}{2-x}\right)^{1/2}$ (4) $\log\left(\frac{x-1}{3-x}\right)^{1/2}$

Q.94 If $f : R \to R$, $f(x) = e^x$ & $g : R \to R$, $g(x) = 3x - 2$, then the value of $(fog)^{-1}(x)$ is equal to -

(1) $\log(x - 2)$ (2) $\frac{2+\log x}{3}$

(3) $\log\left(\frac{x+3}{2}\right)$ (4) None of these

Q.95 If $f(x) = \{4 - (x - 7)^3\}^{1/5}$, then its inverse is-

(1) $7 - (4 + x^5)^{1/3}$

(2) $7 - (4 - x^5)^{1/3}$

(3) $7 + (4 - x^5)^{1/3}$

(4) None of these

Q.96 The inverse of the function $f(x) = \frac{e^x-e^{-x}}{e^x+e^{-x}}$ is

(1) $\frac{1}{2}\ \ell n\frac{1+x}{1-x}$ (2) $\frac{1}{2}\ \ell n\frac{2+x}{2-x}$

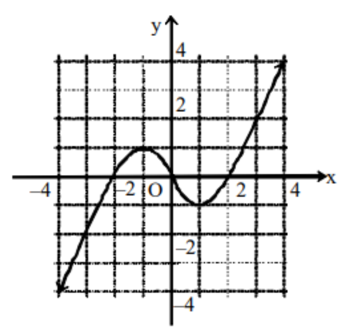
(3) $\frac{1}{2}\ \ell n\frac{1-x}{1+x}$ (4) $2\ \ell n(1 + x)$

TRANSFORMATION OF GRAPH

Q.97 Number of solution of $6\,|\cos x| = x$ in $[0. 2\pi]$ is

(1) 1 (2) 2 (3) 3 (4) 4

Q.98 The graph of the function y = g(x) is shown. The number of solutions of the equation $||g(x)|-1| = \frac{1}{2}$, is



(1) 4 (2) 5 (3) 6 (4) 8

EXERCISE-II ANALYTICAL QUENTIONS

Q.1 If $f : R \to R$, $f(x) = x^3 + 3$, and $g : R \to R$, $g(x) = 2x + 1$, then $f^{-1}og^{-1}(23)$ equals-
(1) 2 (2) 3 (3) $(15)^{1/3}$ (4) $(14)^{1/3}$

Q.2 The period of $f(x) = \dfrac{|\sin x| + |\cos x|}{|\sin x - \cos x|}$ is -
(1) 2π (2) π
(3) $\pi/2$ (4) None of these

Q.3 The function $f(x) = \dfrac{\sec^{-1} x}{\sqrt{x - [x]}}$, where [x] denotes the greatest integer less than or equal to x, is defined for all x belonging to -
(1) R
(2) $R - \{(-1, 1) \cup \{n : n \in Z\}\}$
(3) $R^+ - (0, 1)$
(4) $R^+ - \{n : n \in N\}$

Q.4 The interval for which $\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x} = \dfrac{\pi}{2}$ holds-
(1) [0, 3] (2) $[0, \infty)$
(3) [0, 1] (4) [0, 2]

Q.5 The function $f(x) = \cos^{-1}\left(\dfrac{|x|-3}{2}\right) + [\log_e(4 - x)]^{-1}$ is defined for -
(1) $[-1, 0] \cup [1, 5]$
(2) $[-5, -1] \cup [1, 4]$
(3) $[-5, -1] \cup ([1, 4) - \{3\})$
(4) $[1, 4] - \{3\}$

Q.6 The range of $f(x) = \sin^{-1}\sqrt{x^2 + x + 1}$ is -
(1) $(0, \pi/2]$ (2) $(0, \pi/3]$
(3) $[\pi/3, \pi/2]$ (4) $[\pi/6, \pi/3]$

Q.7 If $f(x) = \dfrac{1}{x+1}$ and $g(x) = \dfrac{1}{\sqrt{x}-1}$, then common domain of function is -
(1) $\{x \mid x < 1, x \in R\}$
(2) $\{x \mid x \geq 0, x \neq 1, x \in R\}$
(3) $\{-1\}$
(4) $\{1\}$

Q.8 If $f(x) = \left(\dfrac{x}{1-|x|}\right)^{1/12}$, $x \in R$ then domain of the function f(x) is -
(1) $(-1, 0]$ (2) $(-\infty, -1) \cup [0, 1)$
(3) $(-1, \infty) - \{1\}$ (4) None of these

Q.9 If $f : R \to R$, $f(x) = \tan x$, then pre-image of -1 under f is-
(1) $\left\{n\pi - \dfrac{\pi}{4} \middle| n \in I\right\}$ (2) $\left\{n\pi + \dfrac{\pi}{4} \middle| n \in I\right\}$
(3) $\{n\pi \mid n \in I\}$ (4) None of these

Q.10 The domain of
$$f(x) = \sqrt{[\cos(\sin x)]} + (1 - x)^{-1} + \sin^{-1}\left(\frac{x^2+1}{2x}\right)$$
equal to -
(1) $(1, \infty)$ (2) $\{-1\}$
(3) $R - \{1\}$ (4) None of these

Q.11 Function $f : R \to R^+$, $f(x) = x^2 + 2$ & $g : R^+ \to R$, $g(x) = \left(1 - \dfrac{1}{1-x}\right)$ then the value of gof (2) is -
(1) 8/7 (2) 5/6
(3) 1/6 (4) 6/5

Q.12 Period of function $2^{\{x\}} + \sin \pi x + 3^{\{x/2\}} + \cos 2\pi x$ is (where { } represent fractional part of x)
(1) 2 (2) 1
(3) 3 (4) None of these

Q.13 Let $f : (4, 6) \to (6, 8)$ be a function defined by $f(x) = x + [x/2]$ where [ ] represent G.I.F. then $f^{-1}(x)$ is equal to -
(1) $x - 2$ (2) $-x - 2$
(3) $x - [x/2]$ (4) None of these

Q.14 If $f(x) = \log\dfrac{1+x}{1-x}$, when $-1 < x_1, x_2 < 1$, then $f(x_1) + f(x_2)$ equals -
(1) $f\left(\dfrac{x_1 + x_2}{1 + x_1x_2}\right)$ (2) $f\left(\dfrac{x_1 + x_2}{1 - x_1x_2}\right)$
(3) $f\left(\dfrac{x_1 - x_2}{1 + x_1x_2}\right)$ (4) $f\left(\dfrac{x_1 - x_2}{1 - x_1x_2}\right)$

Q.15 Period of the function $f(x) = |\sin \pi x| + e^{3(x - [x])}$ (where [ ] represent G.I.F.) is -
(1) 1 (2) 2
(3) 1/3 (4) None of these

Q.16 If period of $\frac{\cos(\sin nx)}{\tan(x/n)}$ $(n \in N)$ is 6π then n is equal to -
(1) 3 (2) 2 (3) 6 (4) 1

Q.17 If [x] and {x} represent the integral and fractional part of x respectively then value of $\sum_{r=1}^{2000} \frac{\{x+r\}}{2000}$ is
(1) x (2) [x]
(3) {x} (4) x + 2001

Q.18 The period of f(x) = cos (sin x) + cos (cos x) is -
(1) $\pi/3$ (2) $\pi/6$ (3) π (4) $\pi/2$

Q.19 If f be the greatest integer function and g be the modulus function, then
$(gof)\left(-\frac{5}{3}\right) - (fog)\left(-\frac{5}{3}\right) =$
(1) 1 (2) –1 (3) 2 (4) 4

Q.20 The domain of function f(x) = log |log x| is-
(1) $(0, \infty)$ (2) $(1, \infty)$
(3) $(0, 1) \cup (1, \infty)$ (4) $(-\infty, 1)$

Q.21 Domain of the function $\tan^{-1} x + \cos^{-1} x^2$ is -
(1) R– [–1, 1] (2) R– (–1, 1)
(3) (–1, 1) (4) [–1, 1]

Q.22 If the domain of function $f(x) = x^2 - 6x + 7$ is $(-\infty, \infty)$, then the range of function is -
(1) $(-\infty, \infty)$ (2) $[-2, \infty)$
(3) (–2, 3) (4) $(-\infty, -2)$

Q.23 Period of $f(x) = \sin 3\pi \{x\} + \tan \pi [x]$ where [ ] and {} represent of G.I.F and fractional part of x
(1) 1 (2) 2 (3) 3 (4) π

Q.24 If S be the set of all triangles and $f : S \to R^+$, $f(\Delta)$ = Area of Δ, then f is -
(1) One-one onto (2) one-one into
(3) many-one onto (4) many-one into

Q.25 If $f : C \to R$, $f(z) = |z|$, then f is -
(1) one-one but not onto
(2) onto but not one-one
(3) neither one-one nor onto
(4) both one-one and onto

Q.26 Which of the following functions are equal?
(1) $f(x) = x$, $g(x) = \sqrt{x^2}$
(2) $f(x) = \log x^2$, $g(x) = 2 \log x$
(3) $f(x) = 1$, $g(x) = \sin^2 x + \cos^2 x$
(4) $f(x) = x/x$, $g(x) = 1$

Q.27 $f : N \to N$ defined by $f(x) = x^2 + x + 1$, $x \in N$ then f is
(1) one-one onto
(2) many-one onto
(3) one-one but not onto
(4) none of these

Q.28 Let $f(x) = \sin^2 (x/2) + \cos^2 (x/2)$ and $g(x) = \sec^2 x - \tan^2 x$. The two function are equal over the set -
(1) R
(2) $R - \left\{x : x = (2n+1)\frac{\pi}{2}, n \in Z\right\}$
(3) ϕ
(4) None of these

Q.29 The domain of the function
$f(x) = \sin^{-1}\left(\frac{2-|x|}{4}\right) + \cos^{-1}\left(\frac{2-|x|}{4}\right) + \tan^{-1}\left(\frac{2-|x|}{4}\right)$ is given by
(1) [0, 6] (2) [–6, 6]
(3) [– 3, 3] (4) None of these

Q.30 The domain of function
$f(x) = \frac{1}{\log_{10}(3-x)} + \sqrt{x+2}$ is -
(1) [–2, 3) (2) [–2, 3) – {2}
(3) [–3, 2] (4) [–2, 3] – {2}

Q.31 Let $f(x) = \frac{\sin([x]\pi)}{x^2 + 2x + 4}$, [.] = G.I.F., then which one is not true -
(1) f is periodic (2) f is even
(3) f is many-one (4) f is onto

Q.32 The domain of function
$f(x) = \log(3x - 1) + 2\log(x + 1)$ is -
(1) $[1/3, \infty)$ (2) $[-1, 1/3]$
(3) $(-1, 1/3)$ (4) None of these

Q.33 If $f(x) = \frac{x}{\sqrt{1+x^2}}$, then (fofof) (x) is equal to-
(1) $\frac{3x}{\sqrt{1+x^2}}$ (2) $\frac{x}{\sqrt{1+3x^2}}$
(3) $\frac{3x}{\sqrt{1-x^2}}$ (4) None of these

Q.34 If f (x) be a polynomial satisfying
f (x). f(1/x)= f (x) + f (1/x) and f(4) = 65 then f(6) = ?
(1) 289 (2) 217
(3) 176 (4) None of these

Q.35 If $f(x) = x^3 - x$ and $g(x) = \sin 2x$, then-
(1) $g[f(1)] = 1$ (2) $f(g(\pi/12)) = -3/8$
(3) $g\{f(2)\} = \sin 2$ (4) None of these

Q.36 Domain of the function $f(x) = \frac{x-3}{(x-1)\sqrt{x^2-4}}$ is -
(1) (1, 2) (2) $(-\infty, -2) \cup (2, \infty)$
(3) $(-\infty,-2) \cup (1, \infty)$ (4) $(-\infty,\infty) - \{1, \pm 2\}$

Q.37 Domain and range of $\sin\left(\log\left(\frac{\sqrt{4-x^2}}{1-x}\right)\right)$ is -
(1) $[-2, 1), (-1, 1)$ (2) $(-2, 1), [-1, 1]$
(3) $(-2, 1)$, R (4) None of these

Q.38 Let $f : R \to R$ be a function defined by
$f(x) = x + \sqrt{x^2}$, then f is-
(1) injective (2) bijective
(3) surjective (4) None of these

Q.39 If $f(x) = e^{3x}$ and $g(x) = \ell n\, x$, $x > 0$, then (fog) (x) is equal to-
(1) 3x (2) x^3
(3) log 3x (4) 3 log x

Q.40 If $f : R \to R$ $f(x) = \cos(5x + 2)$ then the value of $f^{-1}(x)$ is -
(1) $\frac{\cos^{-1}(x)}{5} - 2$ (2) $\cos^{-1}(x) - 2$
(3) $\frac{\cos^{-1}(x) - 2}{5}$ (4) Does not exist

Q.41 $f : R \to R$ is defined by $f(x) = \cos^2 x + \sin^4 x$ for $x \in R$ then the range of f (x) is -
(1) (3/4, 1) (2) [3/4, 1)
(3) [3/4, 1] (4) (3/4, 1)

Q.42 The natural domain of the real valued function defined by $f(x) = \sqrt{x^2-1} + \sqrt{x^2+1}$ is-
(1) $1 < x < \infty$ (2) $-\infty < x < \infty$
(3) $-\infty < x < -1$ (4) $(-\infty,\infty) - (-1, 1)$

Q.43 If $f(x) = \frac{\sqrt{9-x^2}}{\sin^{-1}(3-x)}$, then domain of f is -
(1) [2, 3] (2) [2, 3)
(3) (2, 3] (4) None of these

Q.44 Let $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$ $(x \neq 0)$, then f(x) equals -
(1) $x^2 - 2$ (2) $x^2 - 1$
(3) x^2 (4) None of these

Q.45 Let $f(x) = \sqrt{(2+x-x^2)}$ and
$g(x) = \sqrt{-x} + \frac{1}{\sqrt{x+2}}$. Then domain of f + g is given by -
(1) (0, 1) (2) [0, 1] (3) [–1, 0] (4)(–2 0]

Q.46 $f(x) = \log(\sqrt{x-3} + \sqrt{5-x})$, $x \in R$ then domain of f(x) is
(1) [3, 5] (2) $[-\infty, 3] \cup [5, \infty]$
(3) {3, 5} (4) None of these

Q.47 The range of the function $f(x) = |x-1| + |x-2|$, $-1 \le x \le 3$ is
(1) [1, 3] (2) [1, 5]
(3) [3, 5] (4) None of these

Q.48 The range of the function $y = \log_3(5 + 4x - x^2)$ is
(1) (0, 2] (2) $(-\infty, 2]$
(3) (0, 9] (4) None of these

Q.49 Let $f(x) = \frac{9^x}{9^x + 3}$ and $f(x) + f(1-x) = 1$ then find value of $f\left(\frac{1}{1996}\right) + \left(\frac{2}{1996}\right) + \ldots\ldots + f\left(\frac{1995}{1996}\right)$ is -
(1) 998 (2) 997 (3) 997.5 (4) 998.5

Q.50 The range of

$f(x) = \sqrt{(1-\cos x)\sqrt{(1-\cos x)\sqrt{1-\cos x.......\infty}}}$ is -

(1) [0, 1] (2) [0, 1/2]

(3) [0, 2] (4) None of these

Q.51 The range of $\sin^{-1}[x^2 + 1/2] + \cos^{-1}[x^2 - 1/2]$ where [ ] represent G.I.F.

(1) $\{\pi/2, \pi\}$ (2) $\{\pi/2\}$

(3) $\{\pi\}$ (4) None of these

Q.52 If $x = \log_a bc$, $y = \log_b ca$, and $z = \log_c ab$, then $\frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z}$ equals-

(1) 1 (2) x + y + z

(3) abc (4) ab + bc + ca

Q.53 The range of 5 cos x – 12 sin x + 7 is-

(1) [–6,20] (2) [–3,18]

(3) [–6,15] (4) None of these

Q.54 The domain of the function $\log_2\log_3\log_4(x)$ is-

(1) $(2, \infty)$ (2) $(1, \infty)$

(3) $(3, \infty)$ (4) $(4, \infty)$

Q.55 Let $f(x) = \frac{x-[x]}{1-[x]+x}$, then range of f(x) is ([.] = G.I.F.) -

(1) [0, 1] (2) [1/2, 1]

(3) [0, 1/2] (4) [0, 1/2)

Q.56 The domain of definition of

$f(x) = \sqrt{\log_{0.4}\left(\frac{x-1}{x+5}\right)} \times \frac{1}{x^2-36}$ is –

(1) $(x : x \geq 1, x \neq 6\}$

(2) $(x : x > 0, x \neq 1, x \neq 6\}$

(3) $(x : x > 1, x \neq 6\}$

(4) $(x : x < 0, x \neq -6\}$

Q.57 The function $f : R \to R$ defined by f (x) = (x – 1) (x – 2) (x – 3) is -

(1) one-one but not onto

(2) onto but not one-one

(3) both one and onto

(4) neither one-one nor onto

Q.58 The domain of f(x) is (0, 1) therefore domain of $f(e^x) + f(\ell n\,|x|)$ is -

(1) (–1, e) (2) (1, e)

(3) (–e, – 1) (4) (– e, 1)

Q.59 If $g : [-2, 2] \to R$ where $f(x) = x^3 + \tan x + \left[\frac{x^2+1}{p}\right]$ is a odd function then the value of p where [ ] represent G.I.F. -

(1) – 5< p< 5 (2) p < 5

(3) p > 5 (4) None of these

Q.60 Let $f : R \to R$ be a function defined by

$f(x) = \frac{e^{|x|} - e^{-x}}{e^x + e^{-x}}$. Then -

(1) f is a bijection

(2) f is an injection only

(3) f is a surjection only

(4) f is neither an injection nor a surjection

Q.61 The period of $f(x) = \sin\frac{x}{n!} + \cos\frac{x}{(n+1)!}$ is -

(1) non-periodic

(2) periodic with period (2π) n!

(3) periodic with period 2π (n + 1)!

(4) periodic with period 2 (n + 1) π

Q.62 The function f(x) = max. [1 – x, 1+x, 2] ; $x \in R$ is equivalent to -

(1) $f(x) = \begin{cases} 1-x, x \leq -1 \\ 1, -1 < x < 1 \\ 1+x, x \geq 1 \end{cases}$

(2) $f(x) = \begin{cases} 1+x, x \leq -1 \\ 2, -1 < x < 1 \\ 1-x, x \geq 1 \end{cases}$

(3) $f(x) = \begin{cases} 1-x, x \leq -1 \\ 2, -1 < x < 1 \\ 1+x, x \geq 1 \end{cases}$

(4) None of these

Q.63 The domain of the function $f(x) = {}^{9-x}P_{x-5}$ is-

(1) [5, 7] (2) {5, 6, 7}

(3) {3, 4, 5, 6, 7} (4) None of these

Q.64 The range of the function f(x) = ${}^{9-x}P_{x-5}$ is -

(1) {1, 2, 3} (2) [1, 2]

(3) {1, 2, 3, 4, 5} (4) None of these

Q.65 Domain of the function

$f(x)=\log_2\left(-\log_{1/2}\left(1+\frac{1}{\sqrt[4]{x}}\right)-1\right)$ is-

(1) (0, 1) (2) (0, 1]

(3) [1, ∞) (4) (1, ∞)"

Q.66 The value of n ∈ I for which the function

f(x) = $\frac{\sin nx}{\sin\left(\frac{x}{n}\right)}$ has 4π as its period is-

(1) 3 (2) 4 (3) 2 (4) 5

Q.67 If f(x) is an odd periodic function with period 2, then f (4) equals to-

(1) 0 (2) 4 (3) 2 (4) –4

Q.68 Domain of the function

f(x) = $\sin^{-1}\left(\log_5 \frac{x^2}{5}\right)$ is-

(1) [–5, –1] ∪ [1, 5] (2) [–5, 5]

(3) (–5, –1) ∪ (1, 5) (4) None of these

Q.69 Domain of f(x) = $\sqrt{\frac{1-|x|}{2-|x|}}$ is -

(1) R – [–2, 2]

(2) R – [–1, 1]

(3) [–1, 1] ∪ (–∞, –2) ∪ (2, ∞)

(4) None of these

Q.70 If f(x) = $3\sin\sqrt{\frac{\pi^2}{16}-x^2}$, then values of f(x) lie in

(1) $\left[-\frac{\pi}{4},\frac{\pi}{4}\right]$ (2) [–2, 2]

(3) $\left[0,\frac{3}{\sqrt{2}}\right]$ (4) None of these

Q.71 The period of f(x) = [sin 5x] + |cos 6x| is -

(1) $\frac{\pi}{2}$ (2) 2π (3) π (4) $\frac{2\pi}{5}$

Q.72 Period of f (x) = sin x + tan $\frac{x}{2}$ + sin $\frac{x}{2^2}$ + tan $\frac{x}{2^3}$ + ... + sin $\frac{x}{2^{n-1}}$ + tan $\frac{x}{2^n}$ is -

(1) π (2) 2π (3) $2^n\pi$ (4) $\frac{\pi}{2^n}$

Q.73 The period of f(x) = [x] + [2x] + ... + [nx] – $\frac{n(n+1)}{2}$ x where n ∈ N where [ ] represent G.I.F. is

(1) n (2) 1

(3) $\frac{1}{n}$ (4) None of these

EXERCISE-III PREVIOUS YEAR QUESTIONS

JEE MAINS

Q.1 Which of the following is not a periodic function – **[AIEEE 2002]**

(1) $\sin 2x + \cos x$ (2) $\cos \sqrt{x}$

(3) $\tan 4x$ (4) $\log \cos 2x$

Q.2 The period of $\sin^2 x$ is- **[AIEEE 2002]**

(1) $\pi/2$ (2) π (3) $3\pi/2$ (4) 2π

Q.3 The function $f : R \to R$ defined by $f(x) = \sin x$ is- **[AIEEE-2002]**

(1) into (2) onto

(3) one-one (4) many-one

Q.4 The range of the function $f(x) = \frac{2+x}{2-x}$, $x \neq 2$ is - **[AIEEE-2002]**

(1) R (2) $R - \{-1\}$

(3) $R - \{1\}$ (4) $R - \{2\}$

Q.5 The function $f(x) = \log (x + \sqrt{x^2+1})$, is- **[AIEEE 2003]**

(1) neither an even nor an odd function

(2) an even function

(3) an odd function

(4) a periodic function

Q.6 Domain of definition of the function

$f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$, is- **[AIEEE 2003]**

(1) $(-1, 0) \cup (1, 2) \cup (2, \infty)$

(2) $(1, 2)$

(3) $(-1, 0) \cup (1, 2)$

(4) $(1, 2) \cup (2, \infty)$

Q.7 If $f : R \to R$ satisfies $f(x + y) = f(x) + f(y)$, for all x, $y \in R$ and $f(1) = 7$, then $\sum_{r=1}^{n} f(r)$ is- **[AIEEE 2003]**

(1) $\frac{7n(n+1)}{2}$ (2) $\frac{7n}{2}$

(3) $\frac{7(n+1)}{2}$ (4) $7n(n+1)$

Q.8 A function f from the set of natural numbers to integers defined by

$f(n) = \begin{cases} \frac{n-1}{2}, \text{when } n \text{ is odd} \\ -\frac{n}{2}, \text{when } n \text{ is even} \end{cases}$ is **[AIEEE 2003]**

(1) neither one-one nor onto

(2) one-one but not onto

(3) onto but not one-one

(4) one-one and onto both

Q.9 The range of the function $f(x) = {}^{7-x}P_{x-3}$ is- **[AIEEE 2004]**

(1) {1, 2, 3} (2) {1, 2, 3, 4, 5, 6}

(3) {1, 2,3,4} (4) {1, 2, 3, 4, 5}

Q.10 If $f : R \to S$, defined by $f(x) = \sin x - \sqrt{3}\cos x + 1$, is onto, then the interval of S is- **[AIEEE 2004]**

(1) [0, 3] (2) [–1, 1]

(3) [0, 1] (4) [–1, 3]

Q.11 The graph of the function $y = f(x)$ is symmetrical about the line $x = 2$, then- **[AIEEE 2004]**

(1) $f(x+2) = f(x-2)$ (2) $f(2 + x) = f(2 - x)$

(3) $f(x) = f(-x)$ (4) $f(x) = -f(-x)$

Q.12 The domain of the function $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is- **[AIEEE 2004]**

(1) [2, 3] (2) [2, 3) (3) [1, 2] (4) [1, 2)

Q.13 Let $f : (-1, 1) \to B$, be a function defined by $f(x) = \tan^{-1}\frac{2x}{1-x^2}$, then f is both one-one and onto when B is the interval - **[AIEEE-2005]**

(1) $\left(0, \frac{\pi}{2}\right)$ (2) $\left[0, \frac{\pi}{2}\right)$

(3) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (4) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Q.14 A real valued function f(x) satisfies the functional equation $f(x - y) = f(x) f(y) - f(a - x) f(a + y)$ where a is a given constant and $f(0) = 1$, then $f(2a - x)$ is equal to - **[AIEEE-2005]**

(1) $-f(x)$ (2) $f(x)$

(3) $f(1) + f(a - x)$ (4) $f(-x)$

Q.15 The largest interval lying in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ for which the function $f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)$ defined, is- **[AIEEE 2007]**

(1) $[0, \pi]$ (2) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

(3) $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right)$ (4) $\left[0, \frac{\pi}{2}\right)$

Q.16 Let f : N $\to$ Y be a function defined as f(x) = 4x + 3 where Y = |y $\in$ N : y = 4x + 3 for some x $\in$ N|. inverse of f is - **[AIEEE 2008]**

(1) $g(y) = 4 + \frac{y+3}{4}$ (2) $g(y) = \frac{y+3}{4}$

(3) $g(y) = \frac{y-3}{4}$ (4) $g(y) = \frac{3y+4}{3}$

Q.17 For real x, let $f(x) = x^3 + 5x + 1$, then - **[AIEEE 2009]**

(1) f is one - one but not onto R

(2) f is onto R but not one - one

(3) f is one - one and onto on R

(4) f is neither one - one nor onto R

Q.18 Let $f(x) = (x + 1)^2 - 1, x \geq -1$

Statement - 1 :

The set $\{x : f(x) = f^{-1}(x)\} = \{0, -1\}$.

Statement - 2 : f is a bijection. **[AIEEE 2009]**

(1) Statement -1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement -1

(2) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement -1.

(3) Statement -1 is true, Statement-2 is false.

(4) Statement -1 is false, Statement-2 is true.

Q.19 The domain of the function

$f(x) = \frac{1}{\sqrt{|x| - x}}$ is : **[AIEEE 2011]**

(1) $(-\infty, \infty)$ (2) $(0, \infty)$

(3) $(-\infty, 0)$ (4) $(-\infty, \infty) - \{0\}$

Q.20 Let f be a function defined by y f(x) = $(x - 1)^2 + 1$, $(x \geq 1)$

Statement - 1 : The set $\{x : f(x) = f^{-1}(x)\} = \{1, 2\}$

Statement - 2 : f is bijection and $f^{-1}(x) = 1 + \sqrt{x-1}, x \geq 1$. **[AIEEE 2011]**

(1) Statement–1 is true, Statement–2 is false.

(2) Statement–1 is false, Statement–2 is true.

(3) Statement–1 is true, Statement–2 is true ; Statement– 2 is a correct explanation for Statement–1.

(4) Statement–1 is true, Statement–2 is true ; Statement– 2 is not a correct explanation for statement–1.

Q.21 If f (x) + 2f $\left(\frac{1}{x}\right)$ = 3x, $x \neq 0$ and S = { x $\in$ R : f (x) = f (–x)}; then S : **[JEE Main 2016]**

(1) contains exactly one element.

(2) contains exactly two elements.

(3) contains more than two elements.

(4) is an empty set.

Q.22 The function $f : R \to \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x) = \frac{x}{1+x^2}$, is : **[JEE Main 2017]**

(1) neither injective nor surjective.

(2) invertible

(3) injective but not surjective.

(4) surjective but not injective.

Q.23 The domain of the definition of the function $f(x) = \frac{1}{4-x^2} + \log_{10}(x^3 - x)$ is **[JEE Main 2019]**

(1) $(-1, 0) \cup (1, 2) \cup (3, \infty)$

(2) $(-2, -1) \cup (-1, 0) \cup (2, \infty)$

(3) $(-1, 0) \cup (1, 2) \cup (2, \infty)$

(4) $(1, 2) \cup (2, \infty)$

Q.24 Let $f(x) = a^x$ (a > 0) be written as $f(x) = f_1(x) + f_2(x)$, where $f_1(x)$ is an even function and $f_2(x)$ is an odd function.

Then $f_1(x + y) + f_1(x - y)$ equals **[JEE Main 2019]**

(1) $2f_1(x + y).\ f_2(x - y)$

(2) $2f_1(x + y).\ f_1(x - y)$

(3) $2f_1(x).f_2(y)$

(4) $2f_1(x).f_1(y)$

Q.25 For $x \in \left(0, \frac{3}{2}\right)$, let f(x) = $\sqrt{x}$, g(x) = tan x and h (x) $= \frac{l - x^2}{l + x^2}$ If ϕ(x) = (hof)of)(x), then, $\phi\left(\frac{\pi}{3}\right)$ is equal to **[JEE Main 2019]**

(1) $\tan\frac{\pi}{12}$ (2) $\tan\frac{11\pi}{12}$

(3) $\tan\frac{7\pi}{12}$ (4) $\tan\frac{5\pi}{12}$

Q.26 Let f(x) x^2 , $x \in R$. For any $A \subseteq R$, define g(A) = {x $\in$R: f(x)$\in$A}. If S = [0, 4], then which one of the following statemetns is not true? **[JEE Main 2019]**

(1) f(g(S)) = S (2) g(f(S)) $\neq$ S

(3) g(f(S)) = g(S) (4)f(g(S)) = f(S)

Q.27 Let $\sum_{k=1}^{10} f(a+k) = 16(2^{10} - 1)$, where the function f satisfies f(x + y) = f(x) f(y) for all natural numbers x, y and f(1) = 2. Then, the natural number 'a' is **[JEE Main 2019]**

(1) 2 (2) 4 (3) 3 (4) 16

Q.28 If $f(x) = \left(\frac{1-x}{1+x}\right), |x| < 1$, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to **[JEE Main 2019]**

(1) 2f(x) (2) 2f(x^2)

(3) $(f(x))^2$ (4) –2f (x)

Q.29 For x $\in$R– {0, 1}, let $f_1(x) = \frac{1}{x}$, f_2 (x) = 1 – x and $f_3(x) = \frac{1}{1-x}$ be three given functions. If a function, J(x) satisfies **[JEE Main 2019]**
(f_2 .J.f_1) (x) = f_3 (x), then J(x) is equal to

(1) f_2(x) (2) f_3(x)

(3) f_1(x) (4) $\frac{1}{x}f_3(x)$

Q.30 If the function f : R – {1, –1} $\rightarrow$ A defined by $f(x) = \frac{x^2}{1-x^2}$, is surjective, then A is equal to **[JEE Main 2019]**

(1) R – {–1} (2) [0, ∞)

(3) R – [–1, 0) (4) (0, ∞)

Q.31 Let a function f : (0,∞) $\longrightarrow$ (0,∞) be defined by $f(x) = \left|1 - \frac{1}{x}\right|$. Then, f is **[JEE Main 2019]**

(1) injective only
(2) both injective as well as surjective
(3) not injective but it is surjective
(4) neither injective nor surjective

Q.32 The number of functions f from {1, 2, 3, ...20} onto {1, 2, 3, ..., 20} such that f(k) is a multiple of 3, whenever k is a multiple of 4, is **[JEE Main 2019]**

(1) (15)! × 6! (2) 5^6 × 15

(3) 5! × 6! (4) 6^5 × (15)!

Q.33 Let f : R $\rightarrow$ R be defined by $f(x) = \frac{x}{1+x^2} x \in R$. Then, the range of f is **[JEE Main 2019]**

(1) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (2) (- 1, 1) –{0}

(3) $R - \left[-\frac{1}{2}, \frac{1}{2}\right]$ (4) R – [- 1, 1]

Q.34 Let N be the set of natural numbers and two functions f and g be defined as f, g : N $\longrightarrow$ N such that **[JEE Main 2019]**

$$f(n) = \begin{cases} \frac{n+1}{2}; & \text{if n is odd} \\ \frac{n}{2}; & \text{if n is even} \end{cases}$$

and g(n) = n – $(-1)^n$. Then, fog is

(1) one-one but not onto
(2) onto but not one-one
(3) both one-one and onto
(4) neither one-one nor onto

Q.35 Let A = {x$\in$R : x is not a positive integer}. Define a function f : A $\rightarrow$ R as $f(x) = \frac{2x}{x-1}$, then f is **[JEE Main 2019]**

(1) injective but not surjective
(2) not injective
(3) surjective but not injective
(4) neither injective nor surjective

Q.36 If $g(x) = x^2 + x - 1$ and $g(f(x)) = 4x^2 - 10x - 5$, then find $f\left(\frac{5}{4}\right)$. **[JEE Main 2020]**

(1) $\frac{1}{2}$ (2) $-\frac{1}{2}$ (3) $-\frac{1}{3}$ (4) $\frac{1}{3}$

Q.37 Let $f(x)=\frac{8^{2x}-8^{-2x}}{8^{2x}+8^{-2x}}$ then inverse of f(x) is **[JEE Main 2020]**

(1) $\frac{1}{4}\log_8\left(\frac{1+x}{1-x}\right)$ (2) $\frac{1}{2}\log_8\left(\frac{1-x}{1+x}\right)$

(3) $\frac{1}{4}\log_8\left(\frac{1-x}{1+x}\right)$ (4) $\frac{1}{2}\log_8\left(\frac{1+x}{1-x}\right)$

Q.38 Let $f(x)=\frac{x[x]}{x^2+1}$: (1.3) → f then range of f(x) is (where [.] denotes greatest integer function) **[JEE Main 2020]**

(1) $\left(0,\frac{1}{2}\right)\cup\left(\frac{3}{5},\frac{7}{5}\right]$

(2) $\left(\frac{2}{5},\frac{1}{2}\right)\cup\left(\frac{3}{5},\frac{4}{5}\right]$

(3) $\left(\frac{2}{5},1\right)\cup\left(1,\frac{4}{5}\right]$

(4) $\left(0,\frac{1}{3}\right)\cup\left(\frac{2}{5},\frac{4}{5}\right]$

Q.39 Find the number of Sol. of $\log_{1/2}|\sin x|=2-\log_{1/2}|\cos x|, x\in[0,2\pi]$ **[JEE Main 2020]**

(1) 2 (2) 4 (3) 6 (4) 8

Q.40 The domain of the function f(x) = sin-1 $\left(\frac{|x|+5}{x^2+1}\right)$ is $(-\infty, a)\cup[a,\infty)$. Then a is equal to: **[JEE Main 2020]**

(1) $\frac{1+\sqrt{17}}{2}$ (2) $\frac{\sqrt{17}}{2}+1$

(3) $\frac{\sqrt{17}-1}{2}$ (4) $\frac{\sqrt{17}}{2}$

Q.41 Let f : R R be a function which satisfies $f(x + y) = f(x) + f(y)\ \forall\ x, y \in R$. If f(l) = 2 and $g(n)\sum_{k=1}^{(n-1)}f(k), n\in N$ then the value of n, of which g(n) = 20, is; **[JEE Main 2020]**

(1) 20 (2) 9 (3) 5 (4) 4

Q.42 Let [t] denote the greatest integer ≤ t. Then the equation in x, $[x]^2 + 2[x + 2] - 7 = 0$ has: **[JEE Main 2020]**

(1) exactly two Sol.
(2) infinitely many Sol.
(3) exactly four integral Sol.
(4) no integral Sol.

Q.43 If f(x + y) = f(x) f(y) and $\sum_{x=1}^{\infty}f(x)=2, x,y\in N$ where N is the set of all natural numbers, then the value of $\frac{f(4)}{f(2)}$ is : **[JEE Main 2020]**

(1) $\frac{1}{9}$ (2) $\frac{4}{9}$ (3) $\frac{1}{3}$ (4) $\frac{2}{3}$

Q.44 For a suitably chosen real constant a, let a function, $f : R - \{-a\} \to R$ be defined by $f(x)=\frac{a-x}{a+x}$. Further suppose that for any real number $x \neq -a$ and $f(x) \neq -a$, (fof) (x) = x. Then $f\left(-\frac{1}{2}\right)$ is equal to **[JEE Main 2020]**

(1) –3 (2) $\frac{1}{3}$ (3) $-\frac{1}{3}$ (4) 3

Q.45 Suppose that a function $f: R \to R$ satisfies f(x + y) = f(x) f(y) for all x, y ∈ R and f (l) = 3. If $\sum_{i=1}^{n}f(i)=363$, then n is equal to **[JEE Main 2020]**

(1) 5 (2) 10 (3) 15 (4) 20

Q.46 Let $f : R \to R$ be defined as f(x) = 2x - 1 and g : R - {1} → R be defined as $g(x)=\frac{x-\frac{1}{2}}{x-1}$. Then the composition function f(g(x)) is : **[JEE Main 2021]**

(1) both one-one and onto
(2) onto but not one-one
(3) neither one-one nor onto
(4) one-one but not onto

Q.47 If $a + \alpha = 1, b + \beta = 2$ and $a f(x) + \alpha f\left(\frac{1}{x}\right)=bx+\frac{\beta}{x}$, $x \neq 0$, then the value of the expression $\frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}$ is **[JEE Main 2021]**

(1) 1 (2) 2 (3) 3 (4) 4

Q.48 Let f, g : N $\rightarrow$ N such that f(n + 1) = f(n) + f(a) $\forall$ n $\in$N and g be any arbitrary function. Which of the following statements is NOT true?

[JEE Main 2021]

(1) is one-one
(2) If fog is one-one, then g is one-one
(3) If g is onto, then fog is one-one
(4) If f is onto, then f(n) = $\forall$ n $\in$N

Q.49 Let x denote the total number of one-one functions from a set A with 3 elements to a set B with 5 elements and y denote the total number of one-one functions from the set A to the set A × B .

Then : **[JEE Main 2021]**

(1) y = 273x (2) 2y = 91x
(3) y = 91x (4) 2y = 91x

Q.50 A function f(x) is given by $f(x)=\frac{5^x}{5^x+5}$, then the sum of the series

$$f\left(\frac{1}{20}\right)+f\left(\frac{2}{20}\right)+f\left(\frac{3}{20}\right)+...+f\left(\frac{39}{20}\right)$$

is equal to : **[JEE Main 2021]**

(1) $\frac{19}{2}$ (2) $\frac{49}{2}$
(3) $\frac{39}{2}$ (4) $\frac{29}{2}$

Q.51 Let A = {1, 2, 3 …. , 10} and f : A $\rightarrow$ A be defined as

$$f(k)=\begin{cases}k+1 & \text{if k is odd}\\ k & \text{if k is even}\end{cases}.$$

Then the number of possible functions g : A $\rightarrow$ A such that gof = f is : **[JEE Main 2021]**

(1) 10^5 (2) ${}^{10}C_5$ (3) 5^5 (4) 5!

Q.52 Let f(x) = $\sin^{-1}$ x and $g(x)=\frac{x^2-x-2}{2x^2-x-6}$. If g(2) = $\lim_{x\to 2}$ g(x), then the domain of the function fog is :

[JEE Main 2021]

(1) $(-\infty,-2]\cup\left[-\frac{4}{3},\infty\right)$
(2) $(-\infty,-1]\cup[2,\infty)$
(3) $(-\infty,-2]\cup[-1,\infty)$
(4) $(-\infty,-2]\cup\left[-\frac{3}{2},\infty\right)$

Q.53 The inverse of y =$5^{\log x}$ is : **[JEE Main 2021]**

(1) x = $(1/y)^{\log 5}$ (2) $x=y^{\frac{1}{\log 5}}$
(3) x = $5^{\log y}$ (4) $x=5^{\frac{1}{\log y}}$

Q.54 The real valued function $f(x)=\frac{\cos^{-1}x}{\sqrt{x-[x]}}$, where [x] denotes the greatest integer less than or equal to x, is defined for all x belonging to:

[JEE Main 2021]

(1) all reals except integers
(2) all non-integers except the interval [-1, 1]
(3) all integers except 0, -1, 1
(4) all integers except 0, -1, 1

Q.55 If the functions are defined as $f(x)=\sqrt{x}$ and $g(x)=\sqrt{1-x}$ then what is the common domain of the following functions :
f + g, f – g, f/g, g/f, g – f where
e (f ± g) (x) = f(x) ± g(x),

$(f/g)(x)=\frac{f(x)}{g(x)}$ **[JEE Main 2021]**

(1) $0\le x\le 1$ (2) $0\le x\le 1$
(3) (0,1) (4) (0, 1]

Q.56 Let f : R {3} $\rightarrow$ R – {1} be defined by $f(x)=\frac{x-2}{x-3}$. Let g : R $\rightarrow$ R be given as g(x) = 2x - 3. Then, the sum of all the values of for which $f^{-1}(x)+g^{-1}(x)=\frac{13}{2}$ is equal to

[JEE Main 2021]

(1) 7 (2) 2 (3) 5 (4) 3

Q.57 Let f : R $\left\{\frac{\alpha}{6}\right\}$ →R be defined by $f(x)=\frac{5x+3}{6x-\alpha}$. Then the value of α for which (fof) (x) = x, for all $x\in R-\left\{\frac{\alpha}{6}\right\}$, is : **[JEE Main 2021]**

(1) 6 (2) 8
(3) 5 (4) No such α exists

Q.58 If the domain of the function $f(x)\frac{\cos^{-1}\sqrt{x^2-x+1}}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$ in the interval [α, β), then $\alpha+\beta$ is equal to : **[JEE Main 2021]**

(1) $\frac{3}{2}$ (2) 2] (3) $\frac{1}{2}$ (4) 1

Q.59 Let A = {0, 1, 2, 3, 4, 5, 6, 7}. Then the number of bijective functions $f : A \to A$ such that $f(1) + f(2) = 3 - f(3)$ is equal to **[JEE Main 2021]**

(1) 420 (2) 520 (3) 620 (4) 720

Q.60 Consider function $f : A \to B$ and $g : B \to C$ (A, B, $C \subseteq R$) such that (gof)-1 exists, then : **[JEE Main 2021]**

(1) f and g both are one-one
(2) f and g both are onto
(3) f is one-one and g is onto
(4) is onto and g is one-one

Q.61 Let $f : R \to R$ be defined as $f(x + y) + f(x - y) = 2 f(x) f(y)$, $f\left(\frac{1}{2}\right) = -1$. Then, the value of

$$\sum_{k=1}^{20} \frac{1}{\sin(k)\sin(k+f(k))}$$ is equal to : **[JEE Main 2021]**

(1) $\text{cosec}^2 (21) \cos(20) \cos(2)$
(2) $\sec^2 (1) \sec(21) \cos(20)$
(3) $\text{cosec}^2 (1) \text{cosec}(21) \sin(20)$
(4) $\sec^2 (21) \sin(20) \sin(2)$

Q.62 Let [x] denote the greatest integer $\le x$, where $x \in R$. If the domain of the real valued function $f(x) = \sqrt{\frac{|[x]|-2}{|[x]|-3}}$ is $(-\infty, a) \cup [b, c) \cup [4, \infty)$, a < b < c, then the value of a + b + c is : **[JEE Main 2021]**

(1) 8 (2) 1 (3) –2 (4) –3

JEE ADVANCED

Q.63 Let $f(x) = x^2$ and $g(x) = \sin x$ for all $x \in R$. Then the set of all x satisfying (f o g o g of) (x) = (g o g of) (X), where (f o g) (x) = f(g(x)), is **[JEE-Adv. 2011]**

(1) $\pm\sqrt{n\pi}$, $n \in \{0, 1, 2, ...\}$
(2) $\pm\sqrt{n\pi}$, $n \in \{1, 2, ...\}$
(3) $\frac{\pi}{2} + 2n\pi$, $n \in \{....., -2, -1, 0, 1, 2 ....\}$
(4) $2n\pi$, $n \in \{......, -2, -1, 0, 1, 2, ...\}$

Q.64 The function $f : [0, 3] \to [1, 29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is **[JEE-Adv. 2012]**

(1) one-one and onto.
(2) onto but not one-one
(3) one-one but not onto.
(4) neither one-one nor onto.

Q.65 Let f: $(-1, 1) \to R$ be such that $f(\cos 4\theta) = \frac{2}{2-\sec^2\theta}$ for $\theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$. Then the value(s) for $f\left(\frac{1}{3}\right)$ is (are) **[JEE-Adv. 2012]**

(1) $1-\sqrt{\frac{3}{2}}$ (2) $1+\sqrt{\frac{3}{2}}$
(3) $1-\sqrt{\frac{2}{3}}$ (4) $1+\sqrt{\frac{2}{3}}$

Q.66 Let f: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R$ be given by $f(x) = (\log (\sec x + \tan x))^3$ Then **[JEE-Adv. 2014]**

(1) f(x) is an odd function
(2) f(x) is a one-one function
(3) f(x) is an onto function
(4) f(x) is an even function

Q.67 Let $f(x) = \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$ for all $x \in R$ and $g(x) = \frac{\pi}{2}\sin x$ for all $x \in R$. Let (fog) (x) denote f(g(x)) and (gof) (x) denote g(f(x)). Then which of the following is (are) true? **[JEE-Adv. 2015]**

(1) Range of f is $\left[-\frac{1}{2}, \frac{1}{2}\right]$
(2) Range of fog is $\left[-\frac{1}{2}, \frac{1}{2}\right]$
(3) $\lim\limits_{x \to 0} \frac{f(x)}{g(X)} = \frac{\pi}{6}$
(4) There is an $x \in R$ such that (gof)(x) = 1

Q.68 Let X be a set with exactly 5 elements and Y be a set with exactly 7 elements. If α is the number of one - one functions from X to Y and β is the number of onto functions from Y to X, then the value of $\frac{1}{5!}(\beta - \alpha)$ is .... **[JEE-Adv. 2018]**

Q.69 Let E_1 $\left\{x \in R : x \neq 1 \text{ and } \frac{x}{x-1} > 0\right\}$ and

$E_2 = \left\{x \in E_1 : \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right) \text{is real number}\right\}$.

(Here, the inverse trigonometric function $\sin^{-1}x$ assumes values in $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$) Let $f : E_1 \to R$ be the function defined by $f(x) = \log_e\left(\frac{x}{x-1}\right)$ and $g : E_2 \to R$ be the function defined by $g(x) = \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right)$. **[JEE-Adv. 2018]**

List - I

P. The range of f is

Q. The range of g contains

R. The domain of f contains

S. The domain of g is

1. $\left(-\infty,\frac{1}{1-e}\right] \cup \left[\frac{e}{e-1},\infty\right)$

2. (0,1)

3. $\left[-\frac{1}{2},\frac{1}{2}\right]$

4. $(-\infty,0) \cup (0,\infty)$

5. $\left(-\infty,\frac{e}{e-1}\right]$

6. $(-\infty,0) \cup \left(\frac{1}{2},\frac{e}{e-1}\right]$

The correct option is :

(1) $P \to 4; Q \to 2; R \to 1; S \to 1$

(2) $P \to 3; Q \to 3; R \to 6; S \to 5$

(3) $P \to 4; Q \to 2; R \to 1; S \to 6$

(4) $P \to 4; Q \to 3; R \to 6; S \to 5$

Q.70 For non-negative integers n, let

$$f(n) = \frac{\sum_{k=0}^{n} \sin\left(\frac{k+1}{n+2}\pi\right)\sin\left(\frac{k+2}{n+2}\pi\right)}{\sum_{k=0}^{n} \sin^2\left(\frac{k+1}{n+2}\pi\right)}$$

Assuming $\cos^{-1}x$ takes values in $[0,\pi]$, which of the following options is/are correct ? **[JEE-Adv. 2019]**

(1) $\sin(7\cos^{-1} f(5))=0$

(2) If $\alpha = \tan(\cos^{-1} f(6))$, then $\alpha^2 + 2\alpha - 1 = 0$

(3) $\lim_{n\to 0} f(n) = \frac{1}{2}$

(4) $f(4) = \frac{\sqrt{3}}{2}$

Q.71 Let $f(x) = \sin(\pi\cos x)$ and $g(x) = \cos(2\pi \sin x)$ be two functions defined for $x > 0$. Define the following sets whose elements are written in the increasing order :

$X = \{x : f(x) = 0\}$,
$Y = \{x : f'(x) = 0\}$,
$Z = \{x : g(x) = 0\}$,
$W = \{x : g'(x) = 0\}$,

List -I contains the sets X, Y, Z and W. List-II contains some information regarding these sets.

List-I	**List-II**
(I) X	(P) $\supseteq \left\{\frac{\pi}{2},\frac{3\pi}{2},4\pi,7\pi\right\}$
(II) Y	(Q) an arithmetic progression
(III) Z	(R) NOT an arithmetic progression
(IV)W	(S) $\supseteq \left\{\frac{\pi}{6},\frac{7\pi}{6},\frac{13\pi}{6}\right\}$
	(T) $\supseteq \left\{\frac{\pi}{3},\frac{2\pi}{3}\pi\right\}$
	(U) $\supseteq \left\{\frac{\pi}{6},\frac{3\pi}{4}\right\}$

Which of the following is the only CORRECT combination ? **[JEE-Adv. 2019]**

(1) (III), (P), (Q), (U)
(2) (IV), (P), (R), (S)
(3) (III), (R), (U)
(4) (IV), (Q), (T)

ANSWER KEY

EXERCISE-I

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	3	3	2	2	3	4	1	2	2	4	3	2	4	1	3
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	2	2	2	2	2	2	4	4	3	2	3	2	1	3	2
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	3	2	1	3	4	3	2	1	3	3	3	2	2	3	3
Que.	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	3	4	1	2	1	2	4	3	3	2	3	3	1	2	3
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
Ans.	1	2	2	1	2	3	4	1	2	4	2	2	2	2	2
Que.	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
Ans.	2	3	2	3	4	3	2	3	2	4	3	4	4	1	3
Que.	91	92	93	94	95	96	97	98							
Ans.	3	3	4	2	3	1	3	4							

EXERCISE-II

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	1	2	2	3	3	3	2	2	1	2	4	1	1	1	1
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	3	3	4	1	3	4	2	1	3	3	3	3	2	2	2
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	4	4	2	2	2	2	2	4	2	4	2	4	2	1	3
Que.	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	1	2	2	3	3	2	1	1	4	4	3	2	3	3	4
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73		
Ans.	3	3	2	2	1	4	1	1	3	3	2	3	2		

EXERCISE-III

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	2	2	1,4	2	3	1	1	4	1	4	2	2	4	1	4
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	3	2	2	3	2	2	4	3	4	2	3	3	3	2	3
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	4	1	1	2	1	2	1	2	4	1	3	2	2	4	1
Que.	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	4	2	3	2	3	1	1	3	2	3	3	3	1	4	3
Que.	61	62	63	64	65	66	67	68	69	70	71				
Ans.	3	3	1	2	1,2	1,2,3	1,2,3	119	1	1,2,3	2				

JEE Module Details

(Total = 24)

CLASS - XII : 12 MODULES

PHYSICS

Module - 1	
Ch. No.	Chapter Name
1.	Electrostatics
2.	Capacitor & R-C Circuit
3.	Current Electricity

Module - 2	
Ch. No.	Chapter Name
1.	MEC
2.	Magnetic Materials
3.	Bar Magnets & Earth Magnetism
4.	EMI
5.	AC
6.	EMW

Module - 3	
Ch. No.	Chapter Name
1.	Ray Optics
2.	Wave Optics

Module - 4	
Ch. No.	Chapter Name
1.	Modern Physics
2.	Nuclear Physics
3.	Electronics - Semiconductor
4.	Principles of Communication System

CHEMISTRY

Module -1 (Physical)	
Ch. No.	Chapter Name
1.	The Solid State
2.	Solutions
3.	Electrochemistry
4.	Chemical Kinetics
5.	Surface Chemistry

Module -2 (Inorganic)	
Ch. No.	Chapter Name
1.	The p -Block Elements
2.	General Principles and Processes of Isolation of Elements (Metallurgy)
3.	The d - and f Block Elements
4.	Coordination Compounds

Module -3 (Organic)	
Ch. No.	Chapter Name
1.	Halogen Derivatives
2.	Oxygen Containing Compound
3.	Nitrogen Containing Compound
4.	Biomolecules, Polymers & Chemistry Every Day Life

MATHEMATICS

Module - 1	
Ch. No.	Chapter Name
1.	Functions
2.	Inverse Trigonometric Functions
3.	Matrix
4.	Determinants

Module - 2	
Ch. No.	Chapter Name
1.	Limit
2.	Continuity & Differentiability
3.	MOD
4.	AOD

Module - 3	
Ch. No.	Chapter Name
1.	Integration
2.	Area Under Curve
3.	Differential Equations

Module - 4	
Ch. No.	Chapter Name
1.	Vectors
2.	3 - Dimensional Geometry
3.	Probability

Module - 5	
Ch. No.	Chapter Name
1.	H & D
2.	M. Reasoning
3.	Linear Programing
4.	Statistics

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